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Synthetic money

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Abstract

This paper provides a methodology for constructing synthetic money, which is defined as an optimal currency basket that mimics a single currency. Empirical evidence is provided by constructing a *synthetic dollar* from a currency basket comprised of six currencies that excludes the U.S. dollar. We believe that synthetic money has a number of practical applications, including currency pegging operations by nations, denomination of global bond issues by large firms and countries, and analyses of currency movements over time by interested parties.

JEL classification: C43; C61; F31; G11; G15
Keywords: Synthetic money; Optimal currency basket; Currency invariance

1. Introduction

The main purpose of this paper is to provide a methodology for constructing synthetic money. We define synthetic money as a currency basket whose composite value closely mimics the value of a single currency but does not contain this currency. We begin by discussing some currency index concepts developed by Hovanov, Kolari, and Sokolov (2004) (HKS), including currency invariance and stable

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basket currencies.[1] These currency index concepts are used to solve the problem of finding a currency basket with maximum correlation with the U.S. dollar. We also report empirical results for a synthetic dollar composed of six currencies. Other types of synthetic money could be readily constructed using these methods. Finally, we discuss the practical significance of synthetic money in terms of a number of potential real world applications, including currency pegging, global bonds, and currency analysis.

2. Currency index concepts

In this section we overview currency index concepts in HKS in order to introduce notation and provide the intuition behind the construction of synthetic money. HKS point out that the value of any given currency (e.g., U.S. dollars or USD) can be denominated in various base currencies (e.g., European euros or EURO, British pounds or GBP, and Japanese yen or YEN). Thus, the value of a currency depends on the chosen base currency, which creates ambiguity in the valuation of a currency and introduces well-known difficulties in examining the dynamics of currency values taken at different moments of time (e.g., see Kolari, Sokolov, & Hovanov, 2002). To overcome this base currency problem they proposed a *reduced* (to the moment t_0 — the starting point chosen by the investigator) *normalized value in exchange* of ith currency:

$$\mathrm{RNVal}_i(t/t_0) = \frac{c_{ij}(t)}{\sqrt[n]{\prod_{k=1}^{n} c_{kj}(t)}} \Bigg/ \frac{c_{ij}(t_0)}{\sqrt[n]{\prod_{k=1}^{n} c_{kj}(t_0)}} = \sqrt[n]{\prod_{k=1}^{n} \frac{c_{ik}(t)}{c_{ik}(t_0)}}, \tag{1}$$

where coefficients $c_{ij}(t)$, $i, j=1,\ldots, n$, form a $n \times n$ transitive cross-currency matrix (table) $C(t)=(c_{ij}(t))$ of exchange rates of n currencies at the moment t.

An important implication of the currency invariant index in Eq. (1) is that same value of any particular currency is obtained no matter which base currency j is chosen. This property is advantageous whenever base currency choice leads to ambiguity.

The derivation of an optimal (in any sense) currency basket is problematic, as different base currencies would yield different optimums. Applying Eq. (1), HKS defined a composite currency index in the form of a weighted arithmetical mean, or

$$\mathrm{Ind}(\boldsymbol{w}; t/t_0) = \sum_{i=1}^{n} \boldsymbol{w}_i \mathrm{RNVal}_i(t/t_0), \tag{2}$$

of reduced normalized values in exchange $\mathrm{RNVal}_i(t/t_0)$, $i=1,\ldots, n$, $t=1,\ldots, T$, of the currencies under consideration, with associated optimal *weight-vector* $\boldsymbol{w}=(\boldsymbol{w}_1,\ldots, \boldsymbol{w}_n)$:

$$\sum_{i=1}^{n} \boldsymbol{w}_i = 1^T \boldsymbol{w} = 1, \qquad \boldsymbol{w} \geq 0 \ (\text{where } 1 - n \times 1 \text{ unit column}). \tag{3}$$

HKS show that the currency index $\mathrm{Ind}(\boldsymbol{w}; t/t_0)$ can be viewed as a reduced normalized value in exchange of a complex (composite, aggregate) currency defined by a set (basket) of n simple currencies,

[1] See also Hovanov (2000) and Kolari, Sokolov, Fedotov, and Hovanov (2001).

which are taken in suitable fixed amounts $q_i>0$, $i=1,\ldots,n$.[2] The weight-vector $\boldsymbol{w}$ and corresponding optimal currency amounts q are related by the equality

$$q_i = \mu \boldsymbol{w}_i / c(i,j;t_0), \qquad i=1,\ldots,n, \tag{4}$$

for some positive factor μ (hereafter, for specificity, we will use $\mu=1$, which provides equality $\mathrm{Ind}(\boldsymbol{w};t_0/t_0)=\mathrm{RNVal}_j(t_0/t_0)=1$).

Lastly, HKS apply the composite index in Eq. (2) as a criterion function to the problem of solving for the minimum variance basket of currencies. Optimal weights to minimize the variance of a currency basket can be readily computed using well-known optimization methods for diversifying a portfolio of assets (see Markowitz, 1959). The authors show that this so-called *stable aggregate currency* (or *SAC*) is much more stable over time than other basket currencies or component individual currencies included in the basket.

3. Construction of synthetic money

In this section we apply HKS' currency indices $\mathrm{RNVal}_i(t/t_0)$ and $\mathrm{Ind}(\boldsymbol{w};t/t_0)$ to the problem of constructing a *synthetic currency* (currency basket), which has maximum sample correlation coefficient with a given local $(n+1)$th currency (e.g., the U.S. dollar) not included to the basket. It can be formalized as the following optimization problem:

$$\begin{aligned}\mathrm{corr}(\mathrm{Ind}(\boldsymbol{w}),\mathrm{RNVal}_{n+1}) &= \frac{\mathrm{cov}(\mathrm{Ind}(\boldsymbol{w}),\mathrm{RNVal}_{n+1})}{S(\mathrm{RNVal}_{n+1})S(\mathrm{Ind}(\boldsymbol{w}))} \propto \frac{\sum_{i=1}^{n}\boldsymbol{w}_i \mathrm{cov}(\mathrm{RNVal}_i,\mathrm{RNVal}_{n+1})}{\sqrt{\sum_{i,j=1}^{n}\boldsymbol{w}_i\boldsymbol{w}_j\mathrm{cov}\left(\mathrm{RNVal}_i,\mathrm{RNVal}_j\right)}} \\ &= \frac{\boldsymbol{a}^T\boldsymbol{w}}{\sqrt{\boldsymbol{w}^T\boldsymbol{\Sigma}\boldsymbol{w}}} \xrightarrow[\boldsymbol{w}]{} \max,\end{aligned} \tag{5}$$

under the same restrictions as specified in Eq. (3). Here $\boldsymbol{a}^T=(\mathrm{cov}(\mathrm{RNVal}_1,\mathrm{RNVal}_{n+1}),\ldots,\mathrm{cov}(\mathrm{RNVal}_n,\mathrm{RNVal}_{n+1}))$ is a $1\times n$ vector of sample covariances of the currency indices $\mathrm{RNVal}_1(t/t_0)$ from the basket with the $(n+1)$th currency; $\boldsymbol{\Sigma}=(\mathrm{cov}(\mathrm{RNVal}_i,\mathrm{RNVal}_j))$ is a $n\times n$ sample covariance matrix of currency indices $\mathrm{RNVal}(i;t/t_0)$ included in the synthetic currency basket; and $\mathrm{corr}(\cdot,\cdot)$ and $S(\cdot)$ are the sample correlation coefficient and sample standard deviation, respectively, for the corresponding time series.

In the significant case in which the requirement on non-negativity of weights $\boldsymbol{w}$ is relaxed and the covariance matrix $\boldsymbol{\Sigma}$ is nonsingular, the solution to the optimization problem in Eq. (5) can be found explicitly.

[2] There have been a number of well-known aggregate currencies in practical use in the second part of the 20th century: *EUA* (European Unit of Account), *ECU* (European Currency Unit), *Euro*, *SDR* (Special Drawing Rights), *XAM* (Asian Monetary Unit), *TR* (Transferable Ruble of Comecon), etc.

Indeed, since the restriction $1^T\boldsymbol{w}=1$ is not significant (as any solution of the problem without the restriction can be identified up to a positive multiplier), it can be replaced simply by the requirement $\boldsymbol{w}\neq 0$.

Moreover, as the maximum and minimum in optimization problem (5) (without the restriction $1^T\boldsymbol{w}=1$) is achieved at the same absolute value of the objective function, without loss of generality, one can consider the same maximization problem but for the objective function $\mathrm{corr}^2(\mathrm{Ind}(\boldsymbol{w}), \mathrm{RNVal}_{n+1})$. If the matrix $\boldsymbol{\Sigma}$ is nonsingular, then from general Cauchy–Schwarz inequality one gets:

$$\left(\boldsymbol{a}^T\boldsymbol{w}\right)^2 \leq \left(\boldsymbol{a}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{a}\right)\left(\boldsymbol{w}^T\boldsymbol{\Sigma}\boldsymbol{w}\right), \tag{6}$$

where the equality holds if and only if $\boldsymbol{w}$ is proportional to the vector $\boldsymbol{\Sigma}^{-1}\boldsymbol{a}$.

Hence, one can conclude that the solution to Eq. (5) (which admits the requirement $1^T\boldsymbol{w}=1$) is:

$$\boldsymbol{w}^* = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{a}}{1^T\boldsymbol{\Sigma}^{-1}\boldsymbol{a}}, \tag{7}$$

with optimal correlation coefficient value $\mathrm{corr}(\mathrm{Ind}(\boldsymbol{w}), \mathrm{RNVal}_{n+1}) = \sqrt{\boldsymbol{a}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{a}}/S(\mathrm{RNVal}_{n+1})$.

4. Application to synthetic dollars

Based on currency index methods developed in HKS, we construct a synthetic U.S. dollar currency basket comprised of six major currencies: Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), European euro (EUR), British pound (GBP), and Japanese yen (JPY). Daily exchange rate data are gathered for the period January 1, 2002 to December 31, 2002 (i.e., 251 data points). Table 1 gives the optimal $\boldsymbol{w}^*$ weights (associated with normalized values of the currencies using currency invariance index values) and q^* weights (coinciding to the actual quantities of the currencies in the synthetic dollar currency basket). Correlation coefficients are shown in Table 2.

Fig. 1 rescales our synthetic USD and shows that it closely mimics the USD during the in-sample period January 1, 2002 to December 31, 2002 (i.e., their correlation is 0.99). This result implies that the six different currencies contained in the synthetic USD move up and down in systematic ways, which enables us to build a currency basket that closely tracks the USD. Well-known purchasing power parity and interest rate parity conditions in international trade and finance are likely explanations for the systematic behavior of currency values over time. Similarly, the stability of *SAC* is likely due to currencies' systematic co-movements over time.

Based on the synthetic dollar constructed from 2002 daily data, Fig. 1 also shows out-of-sample results for the period January 1, 2003 to October 16, 2003. Here we see that the synthetic dollar tracks

Table 1
Optimal structure of the synthetic U.S. dollar currency basket

	AUD	CAD	CHF	EUR	GBP	JPY
Optimal weights $\boldsymbol{w}^*$	15.19%	20.64%	13.90%	14.42%	20.40%	15.45%
Optimal values q^*	0.2953	0.3298	0.2282	0.1596	0.1411	20.3934

Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), European euro (EUR), British pound (GBP), and Japanese yen (JPY).

Table 2
Correlation coefficients between RNVal's of simple (AUD, CAD, CHF, EUR, GBP, JPY; USD) and aggregated (Synthetic USD) currencies

	AUD	CAD	CHF	EUR	GBP	JPY	USD	Synthetic USD
AUD	1	0.69	−0.70	−0.69	−0.62	−0.40	0.51	0.52
CAD	0.69	1	−0.98	−0.96	−0.38	−0.12	0.93	0.94
CHF	−0.70	−0.98	1	0.97	0.37	0.05	−0.92	−0.93
EUR	−0.69	−0.96	0.97	1	0.41	−0.02	−0.90	−0.91
GBP	−0.62	−0.38	0.37	0.41	1	−0.28	−0.05	−0.06
JPY	−0.40	−0.12	0.05	−0.02	−0.28	1	−0.18	−0.18
USD	0.51	0.93	−0.92	−0.90	−0.05	−0.18	1	**0.99**
Synthetic USD	0.52	0.94	−0.93	−0.91	−0.06	−0.18	**0.99**	11
Std. deviation	0.0204	0.0312	0.0213	0.0194	0.0130	0.0148	0.0372	0.0018

Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), European euro (EUR), British pound (GBP), and Japanese yen (JPY).

the U.S. dollar closely in the first three months of 2003. Thereafter, the synthetic dollar diverges considerably from the dollar's value over time. We infer the optimal weights for the synthetic dollar need to be periodically revised (e.g., every few months) in order for it to closely follow dollar movements.

5. Potential uses of synthetic money

The creation of synthetic money opens up interesting possibilities for real world applications. For example, consider a country that pegs its currency to the dollar (or euro, etc.) but due to political, cultural, social, or other reasons prefers to peg to a basket currency that does not contain the dollar but whose value moves with the dollar over time. This synthetic dollar could be engineered to

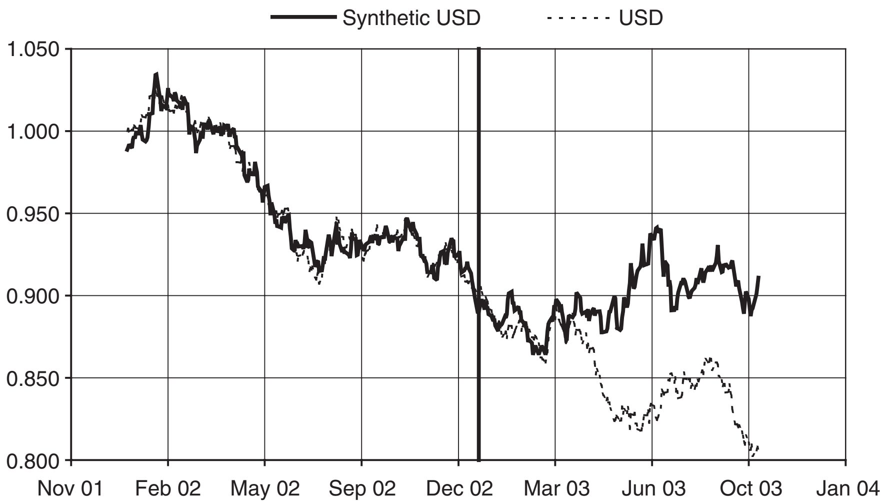

Fig. 1. Dynamics of reduced normalized values for the U.S. dollar, or $\mathrm{RNVal}_{\mathrm{USD}}(t/t_0)$, and the synthetic dollar, or linear positive transformation of $\mathrm{RNVal}_{\mathrm{SyntheticUSD}}(t/t_0)$: In-sample period January 1, 2002 to December 31, 2002 and out-of-sample period January 1, 2003 to October 16, 2003.

closely follow the U.S. dollar as a hard peg or some threshold correlation coefficient could be specified that is lower (e.g., 0.70) to soften the peg. The latter quasi-synthetic dollar would emulate but not mimic the dollar's movement over time. It should be mentioned that, from an economic standpoint, pegging to a synthetic dollar is the same as pegging to the U.S. dollar, as they will move similarly over time. Therefore, non-economic motives would dominate a country's decision to peg to a synthetic currency.

Another potential application is the issuance of global bonds by large firms and national governments. Investors in some parts of the world may prefer to purchase bonds denominated in synthetic dollars instead of U.S. dollars. Currencies are inevitably nationalistic and investor preferences or political realities may offer the strongest case for synthetic money, albeit synthetic dollars, synthetic euros, synthetic pounds, synthetic yen, etc. If a country issued bonds in synthetic dollars, it would pay holders of the bonds in the basket of currencies (excluding dollars). This may be an advantage over dollars, as countries typically hold foreign currency reserves in different hard currencies. By contrast, if the bonds were denominated in dollars and national reserves of dollars were depleted, the country would have to engage in foreign exchange operations to convert hard currency reserves to dollars to meet debt payments. This activity would incur some transaction costs but more importantly might trigger concern among market participants that the country was experiencing some difficulty in meeting debt payments. In turn, interest rates on government debt could possibly increase. By making debt payments in a basket of currencies, a country could potentially have greater debt payment capacity.

Alternatively, if a country issued dollar denominated bonds, it could use synthetic money to maintain foreign currency reserves (excluding dollars) in the proportions recommended by the synthetic money currency basket. As such, the value of non-dollar currency reserves would rise and fall with the dollar and eliminate the currency risk of exchanging these reserves for dollars to pay dollar debt payments.

Additionally, there is the possibility of using synthetic currencies to study currency movements. As discussed above, in the out-of-sample synthetic dollar results shown in Fig. 1, we see that the dollar diverged from its synthetic currency basket after about three months in 2003. Was this divergence due to a change in foreign exchange policy by the United States at that time? The out-of-sample time series of the synthetic dollar in 2003 gives the value of dollar assuming the relationships between the six currencies in 2002 were maintained in 2003. In this regard, notice the sharp decline in the dollar (relative to the synthetic dollar) in April 2003 and large difference in their values over the subsequent six months. One plausible interpretation of these results is that U.S. policy shifted in March 2003 to allow a lower value of the dollar in world currency markets. A lower dollar value would tend to reduce the large trade deficit and stimulate the slow economy. At that time the Treasury Department had been signaling that it would not be displeased if the dollar declined in value. Some experts commented on this "benign neglect" policy as being as effective as direct intervention in foreign currency markets. For example, some currency traders short sold dollars in an attempt to profit on its expected decline in value. Fig. 1 supports this trading behavior, as the dollar moved lower against the other currencies in the synthetic dollar currency basket in spring 2003. We infer that traders could use synthetic currency values to help them in making decisions about speculation in currencies using short and long positions. Also, hedgers could use synthetic money to determine if they need to use various derivative securities (i.e., futures, forward, and options contracts) to offset changes in the value of a currency held in a cash position. Thus, synthetic money provides new ways for policy makers, market observers, traders, and hedgers to evaluate recent currency movements.

Finally, as suggested by an anonymous reviewer, our empirical results suggest that synthetic money could be applied to forecasting currency values. For example, forecasts of currencies' values contained in a synthetic dollar currency basket could be used to construct a forecast of the future value of the U.S. dollar. Forecast values of different currencies (e.g., euros, pounds, yen, etc.) could be obtained from futures (or forward) contracts in these currencies. Additionally, the forecasted value of the synthetic dollar could be compared to futures contract values of the U.S. dollar. If there is a difference between the forecasted synthetic dollar value and futures U.S. dollar value, it may suggest that the futures contracts are over- or undervalued. In this regard, whether forecasts of synthetic currency values are more accurate than forecasts of the currency itself is an empirical question. Of course, if this were true, these future estimates would be useful to traders and hedgers in foreign currencies as discussed above.

Our list of potential uses is not intended to be exhaustive. It is likely that there are other practical uses of synthetic money, which are left for future research.

6. Conclusions

In this paper we reviewed HKS's currency invariance and optimal currency basket concepts and extended their analyses to the construction of synthetic money. To demonstrate the notion of synthetic money, we empirically derived a synthetic dollar using six major currencies (excluding the dollar). The results showed that our synthetic dollar is highly correlated with the U.S. dollar and could be used as a substitute currency.

Synthetic money has a number of potential real world applications. For example, in currency pegging operations, a country could tie their currency to a synthetic dollar, rather than the U.S. dollar. This possibility may be relevant to China, which currently pegs the yuan to the dollar. Due to concerns among its major trading partners, the Bank of China has been considering an alternative pegging system to a basket of currencies. A synthetic dollar could be constructed with less than perfect correlation with the U.S. dollar (i.e., partially mimicking the dollar). This basket currency would be consistent with China's previous currency policy but provide some flexibility vis-à-vis the dollar/yuan exchange rate. Other implications of synthetic money to the issuance of global bonds and currency movement analyses are possible also. Future research is needed to further explore potential applications of synthetic money.

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