N. V. HOVANOV, J.W. KOLARI, M.V. SOKOLOV[1]

SYNTHETIC MONEY[2]

1. INTRODUCTION

The main purpose of this paper is to provide a methodology for constructing synthetic money. We define synthetic money as a currency basket whose composite value closely mimics the value of a single currency but does not contain this currency. We begin by reviewing two currency indexes proposed by Hovanov, Kolari, and Sokolov (HKS) (2004): (1) a currency invariant index (denoted *NVal*), and (2) an optimal currency basket (or stable aggregate currency denoted *SAC*) that has minimum variance for a fixed set of currencies and for a fixed period of time. These currency index concepts are used to solve the problem of finding a currency basket with maximum correlation with the U.S. dollar. We also report empirical results for a synthetic dollar composed of six currencies. Other types of synthetic money could be readily constructed using these methods. Finally, we discuss the practical significance of synthetic money in terms of a number of potential real world applications, including currency pegging, global bonds, and currency analysis.

2. CURRENCY INVARIANCE

Consider a simple exchange model wherein for any pair (g_i, g_j) of currencies g_i, g_j, taken from a fixed set $G = \{g_1, ..., g_n\}$, currency g_i may be exchanged for currency g_j *directly* (i.e. without the necessity of any medium of exchange in the form of another currency or commodity). This direct exchange of currencies g_i, g_j, is quantitatively defined by a positive *exchange coefficient* $c(i, j)$ (see Kolari *et al.* (2001)). This coef-

1 This paper was under review at the International Review of Economics and Finance at the time of going to press.

2 Acknowledgements
The authors gratefully acknowledge financial support from the Eurasia Foundation, U.S. Information Agency, Russian Foundation of Fundamental Researches, the Center for International Business Studies in the Mays Business School at Texas A&M University, and Utrecht School of Economics at Utrecht University. Helpful comments were received from Julian Gaspar, Jaap Bos, Clemens Kool, Michele Fratianni, Peter Schotman, Michael Koetter, Allard Bruinshoofd, Robert Israel and other participants at the Inaugural Conference (Nul-Lustrum), Utrecht School of Economics, Utrecht University, Netherlands (October 2003).

P. de Gijsel and H. Schenk (eds.), Multidisciplinary Economics, 293–303.

ficient gives the amount of the j^{th} currency that can be exchanged for one unit of i^{th} currency. In other words, a coefficient $c(i,j)$ is a *rate of exchange* of i^{th} currency in relation to the j^{th} currency. Thus, matrix $C = (c(i,j))$ forms a cross-currency table of exchange rates. Assuming that matrix C is transitive (i.e. the equality $c(i,j)\cdot c(j,k) = c(i,k)$ holds among every three currencies $g_i, g_j, g_k \in G$), an exchange coefficient $c(i,j)$ can be represented by the form

$$c(i,j) = \frac{Val(i)}{Val(j)}, \tag{1}$$

where magnitude $Val(i)$ is interpreted as the *value in exchange* of the i^{th} currency, $i = 1,\dots,n$ (HKS (2004)). Eq. (1) determines $Val(i)$, $i = 1,\dots,n$ up to a multiplicative constant, so, without loss of generality, we can assign $Val(i) = c(i,j)$, where the j^{th} currency becomes a *numeraire* (unit of account, standard of value, base currency, etc.). To emphasise the role of a base currency g_j, we use the notation $Val(i) = c(i,j) = Val(i/j)$.

The value in exchange dependence on the choice of a base currency introduces difficulties in examining the dynamics of values $Val(i/j)$, $i = 1,\dots,n$, taken at different moments of time. Suppose that we consider three currencies: U.S. dollar (USD), European euro (EUR), and Japanese yen (YEN). The series USD over time is different using EUR versus YEN as the base currency. Thus, the value of USD is dependent on base currency choice.

To overcome this base currency problem of the function $Val(i/j)$, HKS (2004) proposed a *normalised value in exchange*:

$$NVal(i/j) = \frac{Val(i/j)}{\sqrt[n]{\prod_{r=1}^{n} Val(r/j)}} = \frac{c(i,j)}{\sqrt[n]{\prod_{r=1}^{n} c(r,j)}} = \sqrt[n]{\prod_{r=1}^{n} c(i,r)}\,. \tag{2}$$

The most important property of $NVal(i/j)$ is its independence from the choice of a standard currency g_j: $NVal(i/j) = NVal(i/k)$ for any $j,k \in \{1,\dots,n\}$. Since the normalised value in exchange $NVal(i/j)$ is independent of a standard currency g_j, HKS use the notation $NVal(i) = NVal(i/j)$. The normalised value in exchange $NVal(i)$ can be rewritten as a *reduced* (to the moment t_0) *normalised value in exchange*:

$$RNVal(i;t/t_0) = \frac{NVal(i;t)}{NVal(i;t_0)}. \tag{3}$$

The time t_0 is the starting point chosen by the investigator (e.g., $t_0 = 1$). An important implication of currency invariance is that the same value of any particular currency is obtained no matter which base currency is chosen. This property is advantageous whenever base currency choice leads to ambiguity.

3. MINIMUM VARIANCE CURRENCY BASKETS

The derivation of an optimal (minimum variance) currency basket is problematic because different base currencies would yield different optimums. Applying HKS's currency invariance to solve the problem of constructing an optimal currency basket, a composite currency index can be defined as a weighted arithmetical mean:

$$Ind(w;t) = \sum_{i=1}^{n} w_i \; RNVal(i;t/t_0), \tag{4}$$

with reduced normalised values in exchange $RNVal(i;t/t_0)$, $i = 1,..., n$, $t = 1,..., T$, of the currencies $g_1,..., g_n$, and *weight-vector* $w = (w_1,..., w_n)$ (i.e. non-negative weight-coefficients $w_1,..., w_n$, $w_1 + ... + w_n = 1$) associated with the reduced currency invariant valuations of the *n* currencies.

The currency index $Ind(w;t)$ can be viewed as a measure of exchange value of a complex (composite, aggregate) currency $ACU(q) = ACU(q_1,..., q_n)$ defined by a set (basket) of simple currencies, which are taken from a fixed set $G = \{g_1,..., g_n\}$ in suitable fixed amounts $q_i > 0$, $i = 1,..., n$. There have been a number of well-known aggregate currencies in practical use in the second part of the 20th century: *EUA* (European Unit of Account), *ECU* (European Currency Unit), *Euro*, *SDR* (Special Drawing Rights), *XAM* (Asian Monetary Unit), *TR* (Transferable Ruble of Comecon), etc.

Let's suppose that a value in exchange *Val* (*ACU*(*q*)) of a bundle *ACU*(*q*) of simple currencies is determined by the weighted sum

$$Val(ACU(q)) = \sum_{i=1}^{n} q_i \, Val(i) \tag{5}$$

of values in exchange $Val(i)$, $i = 1,..., n$, of the simple currencies. For a fixed moment *t* and base currency *j* we can rewrite Eq. (5) as:

$$Val(ACU(q);t) = \sum_{i=1}^{n} q_i \; c(i, j;t). \tag{6}$$

As such, we can rewrite Eq. (4) as (see Hovanov (2000)):

$$Ind(w;t) = \frac{Val(ACU(q);t)}{Val(ACU(q);t_0)} \sqrt[n]{\prod_{r=1}^{n} \frac{c(r, j;t_0)}{c(r, j;t)}}, \tag{7}$$

where currency weights *w* are defined by the equalities

$$w_i = \frac{q_i \cdot c(i,j;t_0)}{\sum_{k=1}^{n} q_k \cdot c(k,j;t_0)}, \; i = 1,..., n.$$

The index *Ind* (w; t) is a *reduced* (to the moment t_0) *normalised* (being divided by corresponding geometric mean) *value in exchange of the aggregated currency ACU (q) for the moment t*, or

$$Ind(w;t) = RNVal(ACU(q);t/t_0), \quad (8)$$

where currency weights w and corresponding amounts q are related by equalities

$$q_i = \mu\, w_i / c(i,j;t_0), \; i = 1,...,n, \quad (9)$$

for some positive factor μ (hereafter, for specificity, we will use $\mu = 1$, which provides equality $Ind(w;t_0/t_0) = RNVal(j;t_0/t_0) = 1$).

HKS proceeded to construct an aggregated currency $ACU(q^*)$ with minimal volatility of the corresponding time series $Ind(w;t) = RNVal(ACU(q);t)$ (for a fixed set of goods $G = \{g_1,..., g_n\}$, for given exchange rates $c\,(i, j;\, t)$, $i, j = 1,..., n$, $t = 1,..., T$ and for a fixed period of time $[1,T]$). Volatility of the time series is measured by the sample variance

$$S^2(w) = \mathrm{var}(w) = \frac{1}{T}\sum_{t=1}^{T}[Ind(w;t) - MInd(w)]^2, \quad (10)$$

where

$$MInd(w) = \frac{1}{T}\sum_{t=1}^{T} Ind(w;t) \quad (11)$$

is the sample mean.[3] The optimal weight-vector $w^* = (w_1^*,..., w_n^*)$ is determined by the requirement of variance S^2 (w) = var (w) (or standard deviation S (w)) minimisation, $\min_w \mathrm{var}(w) = \mathrm{var}(w^*) = \mathrm{var}(w_1^*,..., w_n^*)$, with constraints $w_i \geq 0,\ i = 1,...,n,\ w_1 + ... + w_n = 1$.

Hence, the optimisation problem is reduced to the quadratic form S^2 (w) minimisation under linear constraints (which is the standard task of quadratic programming). There exist many numerical methods to solve this problem. Here we use the modified Newton's method embodied in Solver.xla by MS Excel 2000.

The optimal amounts q_1^* ,..., q_n^* of the currencies, which are contained in the *optimal aggregated currency ACU* =* $ACU(q^*)$, $q^* = (q_1^*,..., q_n^*)$, can be found from

3 To make our calculations statistically correct we should assume weak stationarity of multivariate time series $c(i, j;t)$, $i, j = 1,..., n$.

Eq. (9). So, any vector of the currencies' amounts $q = (q_1,..., q_n)$ with components proportional to the numbers $w_i */c(i,j;t_0)$, $i=1,...,n$, determines a required optimal aggregated currency $ACU* = ACU(q*)$ that is associated to *time series* $Ind*(t) = Ind(w*;t) = RNVal(ACU*;t)$, $t=1,...,T$, *with minimal variance*. HKS dubbed this optimal basket currency *SAC* (*Stable Aggregate Currency*).

To illustrate the stability of *SAC* over time, we collected daily exchange rate data from January 1, 1999 to May 30, 2001 (after the euro's introduction) for the European euro (EURO), British pound sterling (GBP), Japanese yen (JPY), and U.S. dollar (USD) (source: Pacific Exchange Rate Service http://fx.sauder.ubc.ca/data.html). These exchange rates are normalised using Eq. (2) and reduced to begin at the value of 1.0 on January 1, 1999 using Eq. (3). Eqs. (10) and (11) are then utilised to solve minimum variance weights *w** (for the reduced normalised currency values) and *q** (for the amount of each currency in its own denomination), which are shown in Table 1.

Table 1. SAC results for four hard currencies: January 1, 1999 to May 30, 2001

Optimal weights	EURO	GBP	JPY	USD	SAC
w*(i)	26.94%	26.70%	23.23%	23.13%	
$c(i,j;t_0)$	1.1668	1.6428	0.0088	1.0000	
q*(i)	0.2694	0.2670	26.3174	0.2314	
Standard deviation (i)	0.0510	0.01980	0.0583	0.0315	0.000689

Note: European euro (EURO), British pound sterling (GBP), Japanese yen (JPY), and U.S. dollar (USD).

Table 1 also shows that, within the in-sample period, *SAC* has a standard deviation that is 28 times smaller than the pound and 73 times smaller than the Euro. Hence, *SAC* is very stable relative to individual major currencies. To further demonstrate the stability of *SAC* over time, Figure 1 plots *SAC* and the International Monetary Fund's *SDR* (i.e. the Special Drawing Right contains the same four major currencies with trade-based weights adjusted every five years) for both the in-sample period January 1, 1999 to May 30, 2001 and the out-of-sample period June 1, 2001 to October 31, 2003.

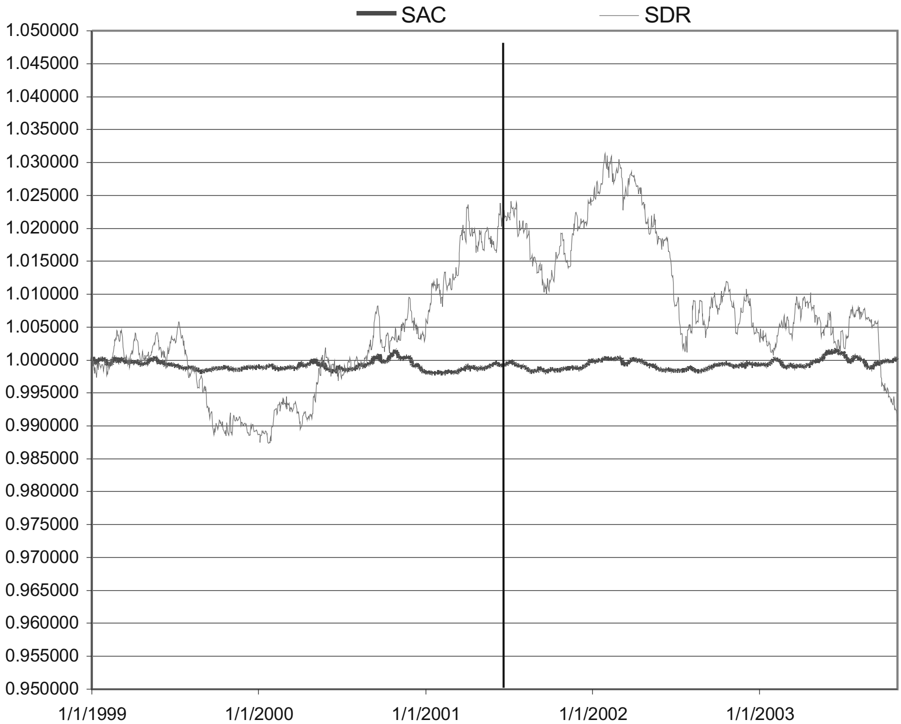

Figure 1. SAC and SDR based on four major currencies over time: In-sample period January 1, 1999 to May 30, 2001 and out-of-sample period June 1, 2001 to October 31, 2003.

The results indicate that *SAC* is more stable than the *SDR* over time (i.e. comparing the in-sample and out-of-sample periods, the standard deviation of *SDR* (SAC) is 0.0092 (0.0007) and 0.0090 (0.0007) respectively). We infer that *SAC* maintains its stability in the out-of-sample period in which the optimal weights from the in-sample period are used for its computation.

4. CONSTRUCTION OF SYNTHETIC MONEY

We next apply HKS' currency indexes *NVal* and *Ind* to the problem of constructing a *synthetic* currency, which has maximum correlation coefficient with given simple currency (e.g. the U.S. dollar). Consider the problem:

$$corr(Ind(w),USD)=\frac{\text{cov}(Ind,USD)}{S(USD)\,S(Ind)}\propto\frac{\sum_{i=1}^{n}w_i\,\text{cov}(i,USD)}{\sqrt{\sum_{i,j=1}^{n}w_iw_j\,\text{cov}(i,j)}}=\frac{a^{\mathrm{T}}w}{\sqrt{w^{\mathrm{T}}\Sigma w}}\xrightarrow[w]{}\max, \quad (12)$$

$$\sum_{i=1}^{n}w_i=\mathbf{1}^{\mathrm{T}}w=1,\ w\geq 0,$$

where $a^{\mathrm{T}}=(\text{cov}(1,USD),...,\text{cov}(n,USD))$ – n x 1 vector of value in exchange covariations of simple currencies with USD; and $\Sigma=(\text{cov}(i,j))$ – n x n value in exchange covariation matrix of currencies contained in the basket.

In the significant case in which the requirement on non-negativity of weights w is relaxed and the covariation matrix Σ is nonsingular, the solution to the optimisation problem in Eq. (12) can be found explicitly.

Indeed, since the restriction $\mathbf{1}^{\mathrm{T}}w=1$ is not significant (as any solution of the problem can be identified up to a positive multiplier), it can be replaced simply by the requirement $w\neq 0$.

Moreover, as the maximum and minimum in optimisation problem (12) (without restriction $\mathbf{1}^{\mathrm{T}}w=1$) is achieved at the same absolute value of the objective function, without loss of generality, one can consider the same maximisation problem but for the objective function $corr^2(Ind(w),USD)$. If the matrix Σ is nonsingular (i.e. positive definite), then from general Cauchy-Schwarz inequality one gets:

$$(a^{\mathrm{T}}w)^2\leq\left(a^{\mathrm{T}}\Sigma^{-1}a\right)\left(w^{\mathrm{T}}\Sigma w\right), \quad (13)$$

where the equality holds if and only if w is proportional to vector $\Sigma^{-1}a$.

Hence, one can conclude that the solution to Eq. (11) (which admits requirement $\mathbf{1}^{\mathrm{T}}w=1$) is:

$$w^*=\frac{\Sigma^{-1}a}{\mathbf{1}^{\mathrm{T}}\Sigma^{-1}a}, \quad (14)$$

with optimal correlation coefficient value $corr(Ind(w),USD)=\sqrt{a^{\mathrm{T}}\Sigma^{-1}a}\big/S(USD)$.

5. APPLICATION TO SYNTHETIC DOLLARS

Based on the solution in the previous section, we develop a synthetic dollar currency basket comprised of six major currencies: Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), European euro (EUR), British pound (GBP), and Japanese yen (JPY). Daily exchange rate data are gathered for the period January 1, 2002 to December 31, 2002 (i.e. 251 data points). The optimal w^* and q^* weights are shown in Table 2. Correlation coefficients are shown in Table 3.

Table 2.Optimal structure of the synthetic U.S. dollar currency basket

	AUD	CAD	CHF	EUR	GBP	JPY
Optimal weights w*(i)	15.19%	20.64%	13.90%	14.42%	20.40%	15.45%
Optimal values q*(i)	0.2953	0.3298	0.2282	0.1596	0.1411	20.3934

Note: Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), European euro (EUR), British pound (GBP), and Japanese yen (JPY).

Table 3. Correlation coefficients between RNVal's of simple and aggregated currencies

	AUD	CAD	CHF	EUR	GBP	JPY	USD	Synthetic USD
AUD	1	0.69	-0.70	-0.69	-0.62	-0.40	0.51	0.52
CAD	0.69	1	-0.98	-0.96	-0.38	-0.12	0.93	0.94
CHF	-0.70	-0.98	1	0.97	0.37	0.05	-0.92	-0.93
EUR	-0.69	-0.96	0.97	1	0.41	-0.02	-0.90	-0.91
GBP	-0.62	-0.38	0.37	0.41	1	-0.28	-0.05	-0.06
JPY	-0.40	-0.12	0.05	-0.02	-0.28	1	-0.18	-0.18
USD	0.51	0.93	-0.92	-0.90	-0.05	-0.18	1	0.99
Synthetic USD	0.52	0.94	-0.93		-0.06	-0.18	0.99	1
Std. Deviation	0.0204	0.0312	0.0213	0.0194	0.0130	0.0148	0.0372	0.0018

Note: Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), European euro (EUR), British pound (GBP), and Japanese yen (JPY).

Figure 2 rescales our synthetic USD and shows that it closely mimics the USD during the in-sample period January 1, 2002 to December 31, 2002 (i.e. their correlation is 0.99). This result implies that the six different currencies contained in the synthetic USD move up and down in systematic ways, which enables us to build a currency basket that closely tracks the USD. Well-known purchasing power parity and interest rate parity conditions in international trade and finance are likely explanations for the systematic behaviour of currency values over time. Likewise, the stability of *SAC* is likely due to currencies' systematic co-movements over time.

Based on the synthetic dollar constructed from 2002 daily data, Figure 2 also shows out-of-sample results for the period January 1, 2003 to October 16, 2003. Here we see that the synthetic dollar tracks the U.S. dollar closely in the first three months of 2003. Thereafter, the synthetic dollar diverges considerably from the dollar's value over time. We infer that the optimal weights for the synthetic dollar need to be periodically revised (e.g. every few months) in order for it to closely follow dollar movements.

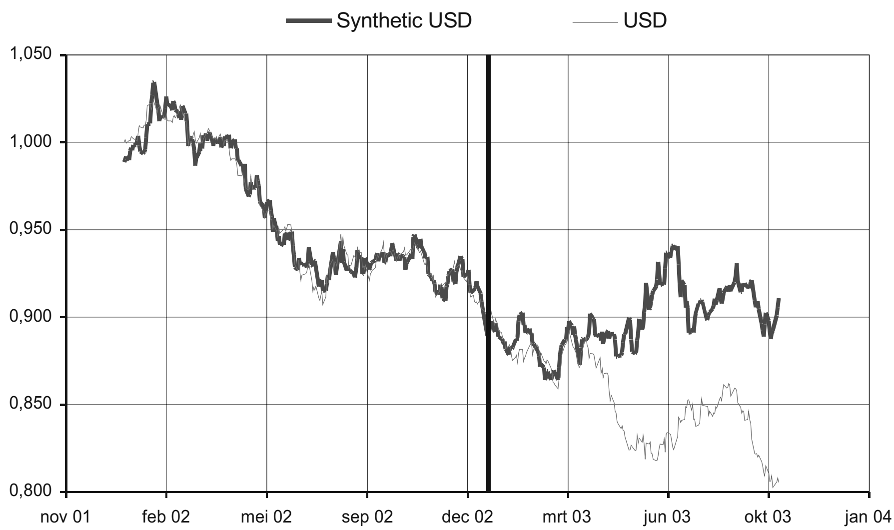

Figure 2.Dynamics of reduced normalised values for the U.S. dollar, or RNVal(USD;t), and the synthetic dollar, or linear positive transformation of RNVal(Synthetic USD;t): In-sample period January 1, 2002 to December 31, 2002 and out-of-sample period January 1, 2003 to October 16, 2003.

6. POTENTIAL USES OF SYNTHETIC MONEY

The creation of synthetic money opens up interesting possibilities for real world applications. For example, consider a country that pegs its currency to the dollar (or euro, etc.) but (for political, cultural, social, or other reasons) would prefer to peg to a basket currency that does not contain the dollar but whose value moves with the dollar over time. This synthetic dollar could be engineered to closely follow the U.S. dollar as a hard peg or some threshold correlation coefficient could be specified that is lower (e.g., 0.70) to soften the peg. The latter quasi-synthetic dollar would emulate but not mimic the dollar's movement over time.

Another potential application is the issuance of global bonds by large firms and national governments. Investors in some parts of the world may prefer to purchase bonds denominated in synthetic dollars instead of U.S. dollars. Currencies are inevitably nationalistic and investor preferences or political realities may offer the strongest case for synthetic money, albeit synthetic dollars, synthetic euros, synthetic pounds, synthetic yen, etc.

Additionally, there is also the possibility of using synthetic currencies to study currency movements. As discussed above, in the out-of-sample synthetic dollar results shown in Figure 2, we see that the dollar diverged from its synthetic currency basket after about three months in 2003. Was this divergence due to a change in foreign exchange policy by the United States at that time? The out-of-sample time series of the synthetic dollar in 2003 gives the value of the dollar assuming the relationships between the six currencies in 2002 were maintained in 2003. In this regard, notice the sharp decline in the dollar (relative to the synthetic dollar) in April 2003 and the large difference in their values over the subsequent six months. One plausible interpretation of these results is that U.S. policy shifted in spring 2003 to allow a lower value of the dollar in world currency markets. A lower dollar value would tend to reduce the large trade deficit and stimulate the slow economy. At that time the Treasury Department had been signalling that it would not be displeased if the dollar declined in value. Some experts commented on this 'benign neglect' policy as being as effective as direct intervention in foreign currency markets (e.g. currency traders sold dollars short in an attempt to profit on its expected decline in value). Thus, the synthetic dollar enables some perspective in evaluating recent dollar movements.

Our list of potential uses is not intended to be exhaustive. It is likely that there are other practical uses of synthetic money, which are left for future research.

7. CONCLUSIONS

In this paper we reviewed HKS's currency invariance and optimal currency basket concepts, illustrated their application to currency data, and extended their analyses to the construction of synthetic money. To demonstrate the notion of synthetic money, we empirically derived a synthetic dollar using six major currencies (excluding the dollar). The results showed that our synthetic dollar is highly correlated with the U.S. dollar and could be used as a substitute currency.

Synthetic money has a number of potential real world applications. For example, in currency pegging operations, a country could tie their currency to a synthetic dollar, rather than the U.S. dollar. This possibility may be relevant to China, which currently pegs the yuan to the dollar. Due to concerns among its major trading partners, the Bank of China has been considering an alternative pegging system to a basket of currencies. A synthetic dollar could be constructed with less than perfect correlation with the U.S. dollar (i.e. partially mimicking the dollar). This basket currency would be consistent with China's previous currency policy but provide some flexibility vis-à-vis the dollar/yuan exchange rate. Other implications of synthetic money to the issuance of global bonds and currency movement analyses are possible also. Future research is needed to further explore potential applications of synthetic money.

REFERENCES

Hovanov, N. (2000). Stable units of account – A base of an informational support in financial economics. *Abstracts of the 7th International Conference on Regional Informatics, St. Petersburg, Russia (December 5-8*) p. 95 (in Russian).

Hovanov, N.V., Kolari, J.W. & Sokolov, M.V. (2004). Computing currency invariant indices with an application to minimum variance currency baskets. *Journal of Economic Dynamics and Control* (forthcoming).

Kolari, J., Sokolov, M., Fedotov, Y. & Hovanov, N. (2001). A simple model of exchange: Indices of value in exchange. *Proceedings of St. Petersburg State University, No. 13,* 141-147 (in Russian).