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Computing currency invariant indices with an application to minimum variance currency baskets

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Abstract

This paper provides an exact and computable *invariant currency value index* (*ICVI*) which is independent of base currency choice. Thus, given a fixed set of currencies, the index of a currency will have the same value, regardless of base currency choice. This currency index can be used as an indicator to assess movements of an individual currency's value in world currency markets. The methodological and mathematical reasoning behind *ICVI* is formulated in terms of a *simple exchange model* (SIMEX).

To demonstrate one possible application we employ *ICVI* to construct a currency basket of minimum variance. Utilizing a quadratic optimization framework, we compute optimal weights for currencies and construct a *stable aggregate currency* (*SAC*). Comparative empirical analyses of a five-currency *SAC* and the IMF's Special Drawing Rights (*SDR*) demonstrates that the *SAC* has lower volatility and lower correlations with its components than the *SDR*. In a similar way it is shown that a three-currency *SAC* has a smaller variance than the world money basket proposed by R. Mundell. Numerous academic and business implications are possible for further study with the use of the indices *ICVI* and *SAC*.

JEL classification: C43; C61; C63; D81; F31; F33; G11; G15

Keywords: Currency index; Currency basket; Minimal variance; Numeraire; International finance; World money

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1. Introduction

It is common practice in international economics and finance to denominate multiple currencies in terms of a base currency or numeraire. One problem with this multi-currency convention is that, depending on the base currency chosen, the resulting time series can dramatically change their dynamics due to fluctuations in currency values over time. For example, the relationship between the yen and pound sterling will be different if the dollar is used as the numeraire from the case when the euro is used as numeraire.

This paper provides an exact and computable *invariant currency value index* (*ICVI*) which is independent of base currency choice. Thus, given a fixed set of currencies, the index of a currency will have the same value, regardless of base currency choice. As such, *ICVIs* for the dollar, euro, and yen are independent of a chosen base currency. The conception of the invariant index of a currency's value in exchange is based on a *simple exchange model* (SIMEX), which describes direct pair-wise exchanges of goods (commodities, services, currencies, etc.). We show that *ICVIs* could be used to better understand the valuation of currencies and other assets in a global context in which multiple currency participants exist. In this respect they could be used to gain insight into currency value questions such as: Did the U.S. dollar go up or down in world currency markets? To our knowledge, no other work has been published on this potentially valuable multi-currency index. *ICVIs* could be applied to a variety of empirical problems in international economics and finance.

To demonstrate one possible application we employ our *ICVI* to solve for a minimum variance currency basket. Throughout history, economists have sought a stable numeraire (or benchmark commodity) for the purpose of international trade and finance. Utilizing a variance minimization framework, we compute optimal weights for five hard currencies and construct a *stable aggregate currency* (*SAC*). Comparative empirical analysis of *SAC* and the IMF's Special Drawing Rights (*SDR*) for the period 1981–1998 demonstrates the low volatility and low correlations with its components of this stable aggregate currency. We therefore conclude that the *SAC* could be used to harden the *SDR*.

Following recent work by Nobel Laureate Robert Mundell, further analyses consider the problem of constructing a stable world currency. In Mundell's words, "A few economists have recently recognized the merits of and need for a world currency. Whether that can be achieved in the near future will depend on politics as well as economics. But it is, nevertheless, a project that would restore a needed coherence to the international monetary system." (Mundell and Friedman, 2001, p. 27). A comparative analysis of Mundell's recommended world currency basket to our minimum variance currency basket constructed from the same currencies confirms the low volatility and correlations with its components of *SAC*. We conclude that the simplicity of *SAC* could be used to develop a world money that would be easy for businesses in the financial services industry to implement.

Our invariant currency value index and stable aggregate currency have major implications to the construction of index numbers for contract settlement. Shiller (1993) has argued that the development of macro markets is dependent on the availability of generally agreed upon indices that are well suited to the settlement of contracts (see Shiller, 1993, p. 208). Price index methods have been applied to a variety of measurement problems, including inflation (i.e., consumer and produced price indices), stock prices (i.e., aggregate market indices), real estate prices (i.e., constant quality indices), etc. In the present paper, $ICVI$ is an index of value that eliminates the problem of base currency. Also, SAC is an index method of constructing minimum variance currency baskets. We believe that these new indices contribute to the effort to create widely agreed upon measures of international currency for use in basket currencies, as well as international prices of financial assets, real goods, and services.

In Section 2 a simple exchange model (SIMEX) is outlined. Section 3 derives our invariant currency value index ($ICVI$). Section 4 applies $ICVI$ to solving for minimum variance currency baskets, which we refer to as a stable aggregate currency (SAC). Section 5 applies SAC to the problem of hardening the IMF's SDR. Section 6 extends the analyses to the problem of stable world money. Section 7 gives conclusions and implications. The Appendix contains further discussion.

2. Simple exchange model

Here we set forth a simple exchange model (SIMEX) and associated notations to be employed in forthcoming sections. To begin, it is supposed that there is a fixed finite set of infinitely divisible goods (e.g., commodities, services, currencies, etc.) $G = \{g_1, \dots, g_n\}$, where the amount of the ith good is a real number $q_i > 0$. The amount of the ith good can be written in the form $q_i[u_i]$, where u_i is the unit of measurement, i.e., $[u_i]$ is a 'dimension' of the amount q_i (see Bridgman, 1932).

Next, suppose that, for any pair of goods g_i, g_j from the set $G = \{g_1, \dots, g_n\}$, good g_i may be exchanged for good g_j directly (i.e., without using a medium of exchange). This direct exchange of goods g_i, g_j can be quantitatively defined by a positive exchange coefficient c_{ij}. This coefficient gives the amount of the jth good which one can exchange for one unit u_i of the ith good. In the case when the goods under exchange are currencies, the coefficient c_{ij} is a rate of exchange of the ith currency for the jth currency.[1] This *simple exchange model* (SIMEX) of pair-wise direct exchanges of any two goods (e.g., commodities, services, currencies, etc.) will be used as a framework for invariant currency index construction (see Hitrov and Hovanov, 1992).

An exchange matrix $C = (c_{ij})$, $i, j = 1, \dots, n$ of exchange coefficients $c_{ij} > 0$ and a fixed finite set of goods $G = \{g_1, \dots, g_n\}$ form a simple market $M = (G, C)$, where goods $g_1, \dots, g_n$ are being exchanged directly in proportions determined by the constant positive exchange coefficients c_{ij}, $i, j = 1, \dots, n$.[2]

[1] It should be noted that the assumption of direct exchangeability of any two goods from the set $G = \{g_1, \dots, g_n\}$ narrows applications of our simple exchange model to commodities' barter (Davies, 1996). However, this assumption does not invoke any loss of generality when considering currency exchange, as any two currencies can normally be directly exchanged.

[2] Only simple goods $g_1, \dots, g_n$ are under exchange. Exchange coefficients for aggregated goods (i.e., baskets of simple goods from the set $G = \{g_1, \dots, g_n\}$) are not addressed.

A good's amount (or an exchange coefficient), which is specified by a real number q on one numerical scale, may be respecified by a real number $q' = \varphi(q)$ on another numerical scale, where φ is a monotone increasing continuous function that maps the set of all real numbers on itself.[3] Here we will restrict ourselves to the simplest case of an increasing homogeneous linear function $q' = \varphi(q) = \alpha q$, $\alpha > 0$. The transformation of the scale's measurement unit from an old unit u to a new unit u' can be expressed as $[u] = \alpha[u']$, $[u'] = 1/\alpha[u]$, where the coefficient α gives the quantity of new units u' (old units u) contained in one old unit u (in one new unit u'). These simple equations imply that an amount of a good represented in units u by the form $q[u]$ can also be represented in units u' by the form $q'[u']$.[4]

We next consider the situation in which an amount q_i of units u_i of the ith good (currency) is exchanged for an amount q_j of units u_j of the jth good (currency). First, the relation of exchangeability $q_j[u_j] \equiv q_i[u_i]$ between ith and jth goods is generally stated as an equivalence relation with reflexive, symmetric, and transitive properties (Kuratowski and Mostowski, 1967). Second, there exists a numerical function $Val(q[u]) = Val(q;[u])$ of a good's amount q (where q is measured by a unit $[u]$) such that $q_i[u_i] \equiv q_j[u_j]$ if and only if $Val(q_i[u_i]) = Val(q_j[u_j])$. The function $Val(q[u])$ can be treated as a measure of Adam Smith's 'value in exchange' ('relative or exchangeable value') of an amount q of a corresponding good (Smith, 1976, Chapter IV).

It is reasonable to suppose that the function $Val(q[u]) = Val(q;[u])$ is an increasing function and meets the following condition of zero starting point: $Val(0[u]) = Val(0;[u]) = 0$. Also, we will suppose that the value in exchange $Val(q;[u])$ is a continuous function of a corresponding good's amount q, and that this function is additive ($Val(q^{(1)} + q^{(2)};[u]) = Val(q^{(1)};[u]) + Val(q^{(2)};[u])$ for all amounts $q^{(1)} \geqslant 0$, $q^{(2)} \geqslant 0$ of a fixed good from the set $G = \{g_1,\dots,g_n\}$.

It is well known that an additive, continuous function $Val(q;[u])$, which meets the above-mentioned condition of zero starting point, has the unique representation $Val(q;[u]) = \alpha q$, where $\alpha \neq 0$ (see Dieudonne, 1960). The additional condition of the function $Val(q;[u])$ increasing implies $\alpha > 0$. Substituting $q = 1$ into the representation determines the constant $\alpha = Val(1;[u]) = Val([u])$. Under these conditions, we obtain the following explicit function:

$$Val(q[u]) = Val(q;[u]) = qVal([u]). \tag{2.1}$$

The function $Val(q;[u])$ can be interpreted as an indicator or *index* of the value in exchange of an amount $q[u]$ for any given good (currency) from the set $G = \{g_1,\dots,g_n\}$.

[3] According to the modern measurement theory, the numerical scale by which amounts of goods are measured is not unique but is given with a precision of homeomorphism of the set of all real numbers, i.e., with a precision of an increasing continuous transformation (e.g., see Dieudonne, 1960; Pfanzagl, 1971; Potter, 1999).

[4] Numerical scales, which are produced from each other by increasing homogeneous linear transformations $q' = \varphi(q) = \alpha q$, $\alpha > 0$, are *scales of ratios* because the ratio $q^{(1)}/q^{(2)}$ of two measured amounts $q^{(1)}$, $q^{(2)}$ is invariant in relation to choosing a variant of the increasing homogeneous linear transformation φ: $\varphi(q^{(1)})/\varphi(q^{(2)}) = q^{(1)}/q^{(2)}$.

For any two goods (currencies) we now can write the ratio

$$\frac{Val([u_i])}{Val([u_j])} = \frac{q_j}{q_i} = c_{ij}, \tag{2.2}$$

which determines element c_{ij} of the exchange matrix $C = (c_{ij})$, $i, j = 1, \ldots, n$. Thus, an observable proportion c_{ij} of exchange can be treated in our simple exchange model as a ratio of two corresponding non-observable values in exchange $Val([u_i])$, $Val([u_j])$ of the goods' (currencies') units. This theoretical interpretation of the empirical data c_{ij}, $i, j = 1, \ldots, n$ in terms of values of the function $Val(q; [u])$ raises the question as to whether there exists a one-argument function $Val(q; [u])$, such that (2.2) holds for all exchange coefficients treated as values of a two-arguments function c_{ij}, $i, j = 1, \ldots, n$. The next proposition provides the solution to this question (for proof, see Hitrov and Hovanov, 1992):

Proposition 1. *All elements of an exchange matrix $C = (c_{ij})$, $c_{ij} > 0$, $i, j = 1, \ldots, n$, may be represented as ratios $c_{ij} = Val([u_i])/Val([u_j])$ of values of a one-argument value function $Val(q; [u])$ if and only if the matrix C is transitive, i.e., the relation $c_{ij}c_{jk} = c_{ik}$ among every three elements c_{ij}, c_{jk}, c_{ik} of the matrix holds.*

In view of this proposition, we will add to our simple exchange model the condition of *transitivity* imposed on the exchange matrix $C = (c_{ij})$. Under this requirement, the following formula holds for any series of indices $i, k_1, \ldots, k_s, j$ of goods (currencies) under exchange:

$$c(i, k_1)^* c(k_1, k_2)^* \cdots {}^* c(k_s, j) = c(i, j). \tag{2.3}$$

This equation states that there is no possibility for an exchange arbitrage between different parts of the simple market $M = (G, C)$ with transitive exchange matrix C. Other implications of transitivity are reflexivity ($c_{ii} = 1$, $i = 1, \ldots, n$) and reciprocal symmetry ($c_{ji} = 1/c_{ij}$, or $c_{ij}c_{ji} = 1$) of the matrix. All columns (rows) of a transitive exchange matrix $C = (c_{ij})$ are proportional to each other. Consequently, the rank of transitive exchange matrix $C = (c_{ij})$, $i, j = 1, \ldots, n$ equals 1. In turn, the maximal eigenvalue of the matrix is equal to n, and its right eigenvector (column) is proportional to a column of the matrix (Horn and Johnson, 1986).

The above-mentioned properties of an exchange matrix $C = (c_{ij})$ may be used for estimation of value in exchange of units $[u_1], \ldots, [u_n]$ of corresponding goods (currencies). For example, we can take elements $Val([u_i]) = c_{ij}$, $i = 1, \ldots, n$, of the jth column of the exchange matrix for these estimates. In this case the jth good (currency) becomes a standard or *base good* (*base currency*) in relation to which values in exchange of all other goods (currencies) are measured. In other words, unit $[u_j]$ becomes a *numeraire* (i.e., *unit of account* or *standard of value*) for measuring value in exchange: $Val([u_j]) = c_{jj} = 1$. To emphasize the role of a standard good (standard currency) g_j, we will adopt the following notation: $Val([u_i]) = c_{ij} = Val([u_i]/[u_j]) = Val_{ij}$.

3. Invariant currency value indices

The dependence of the value in exchange on the base currency (or standard good) choice creates some difficulties in examining the dynamics of values $Val_{ij},\ i=1,\dots,n$. For example, consider cross rates $c_{ij}(t),\ i,j=1,\dots,5$ of the five currencies German mark (DEM), French franc ($FRF'=10\ FRF$), British pound sterling (GBP), Japanese yen ($JPY'=100\ JPY$), and U.S. dollar (USD) for $T=216$ moments of time. These cross rates are gathered from IMF statistical series for months ending from January 31, 1981 until December 31, 1998. Fixing the U.S. dollar ($[u_5]=[USD]$) as the base currency, we can calculate the reduced (to the initial moment $t=1$) value in exchange as $RVal_{i5}(t/1)=c_{i5}(t)/c_{i5}(1)$ for all currencies ($i=1,\dots,5$) and moments of time ($t=1,\dots,216$). The resulting five time series $RVal_{i5}(t/1)$ may be compared with the corresponding five time series $RVal_{i1}(t/1)=c_{i1}(t)/c_{i1}(1)$ using the German mark ($[u_1]=DEM$) as the base currency.

By using different base currencies, the dynamics of the time series of currency values are markedly changed. Moreover, the correlations among currencies FRF, GBP and JPY are quite different for these two numeraire (i.e., USD and DEM). The corresponding correlation coefficients for the time series $RVal_{ij}(t/1),\ i=2,3,4,\ j=1,5$, are shown in Table 1, which contains two panels: panel A (USD numeraire) and panel B (DEM numeraire). The strong positive correlation between reduced values in exchange of the French franc and the Japanese yen using the U.S. dollar as the base currency stands in sharp contrast to the strong negative correlation of these currencies using the German mark as the base currency. Thus, due to the fact that comparisons based on the index Val_{ij} depend on the numeraire chosen, we infer that this index of value in exchange is not an adequate tool for treating the time series $Val_{ij}(t)$ or $RVal_{ij}(t/1)$.

To overcome this drawback we will utilize the fact that value in exchange is measured by a *scale of ratios* with a precision of a positive factor β (e.g., see Dieudonne, 1960; Pfanzagl, 1971; Potter, 1999). We define β as the inverse of the geometric mean of values $Val_{1j},\dots,Val_{nj}$, or

$$GMean(Val_{1j},\dots,Val_{nj})=\left(\prod_{r=1}^{n}Val_{rj}\right)^{1/n}=\sqrt[n]{\prod_{r=1}^{n}Val_{rj}}. \tag{3.1}$$

In effect, a new indicator of value in exchange is introduced in the form of a *normalized value in exchange* (*normalized index of value in exchange*), or

$$NVal_{ij}=\beta Val_{ij}=\frac{Val_{ij}}{GMean(Val_{1j},\dots,Val_{nj})}=\frac{Val_{ij}}{\sqrt[n]{\prod_{r=1}^{n}Val_{rj}}}=\frac{c_{ij}}{\sqrt[n]{\prod_{r=1}^{n}c_{rj}}}. \tag{3.2}$$

An important property of the defined normalized index of value in exchange $NVal_{ij}$ is its invariance in relation to a choice of a base (standard) good. This property is formally stated as follows:

Table 1
Correlation coefficients $corr(i,k)$, $i,k=2,3,4$, for reduced values in exchange of three currencies (French franc (FRF), British pound (GBP), and Japanese yen (JPY)) calculated using monthly data for the period 1/31/81–12/31/98: U.S. dollar (USD) versus German mark (DEM) as the base currency

Panel A. USD base currency			
$i \setminus k$	2. *FRF*	3. *GBP*	4. *JPY*
2. *FRF*	+1.00	+0.62	+0.75
3. *GBP*	+0.62	+1.00	+0.09
4. *JPY*	+0.75	+0.09	+1.00
Panel B. DEM base currency			
$i \setminus k$	2. *FRF*	3. *GBP*	4. *JPY*
2. *FRF*	+1.00	+0.88	-0.69
3. *GBP*	+0.88	+1.00	-0.75
4. *JPY*	-0.69	-0.75	+1.00

Proposition 2. *For a positive homogeneous transformation* $NVal_{ij}=\varphi(Val_{ij})=\beta Val_{ij}$, $\beta>0$, *to be independent of the standard good (currency)* g_j *choice, it is sufficient to fix* β *as the inverse of the geometric mean* $GMean(Val_{1j},\dots,Val_{nj})$) *of the values in exchange* $Val_{ij}=c_{ij}$, $i=1,\dots,n$.

This proposition can be easily proven by explicit calculations. Because the normalized value in exchange $NVal_{ij}$ is independent of the base (standard) good (currency) g_j ($NVal_{ij}=NVal_{il}$) , we will use the notation $NVal_i=NVal_{ij}$. No matter which base currency is employed, the *invariant currency value index* (*ICVI*) $NVal_i$ would have the same value for each base currency (good).[5]

Our *ICVI* $NVal_i$ can be used to investigate empirical time series of currencies' rates of exchange. In this context, instead of using $NVal_i(t)$, it is convenient to employ the reduced (to the moment t_0) normalized value in exchange

$$RNVal_i(t/t_0)=\frac{NVal_i(t)}{NVal_i(t_0)}, \tag{3.3}$$

with starting point t_0 being chosen by the investigator ($RNVal_i(t_0/t_0)=1$ for all $i=1,\dots,n$). Without loss in generality, it can be assumed that $t_0=1$.

Returning to our previous example with time series of cross rates $c_{ik}(t)$, $i,k=1,\dots,5$, $t=1,\dots,216$ of five currencies, the time series of reduced normalized values in exchange $RNVal_i(t/1)$, $i=1,\dots,5$, $t=1,\dots,216$ are calculated. These time series are

[5] It should be recognized that, strictly speaking, the normalized value in exchange $NVal_i$ is dependent on a standard currency, although this standard currency is not an element of the set $G=\{g_1,\dots,g_n\}$ of all simple currencies, which are under exchange in the simple market $M=(G,C)$. Instead, $NVal_i$ utilizes an aggregate (or complex) currency as the standard currency (numeraire). In other words, the standard currency is in this case a basket of all simple currencies $g_1,\dots,g_n$, where the value in exchange of this basket is calculated as a geometric mean $GMean(Val_{1j},\dots,Val_{nj})$ of values in exchange $Val_{1j},\dots,Val_{nj}$ of the simple currencies.

Table 2
Correlation coefficients $corr(i,k)$, $i,k=1,\ldots,5$, for invariant indices (reduced normalized values in exchange) of five currencies (German mark (DEM), French franc (FRF), British pound (GBP), Japanese yen (JPY), and U.S. dollar (USD)) calculated using monthly data for the period 1/31/81–12/31/98

$i \setminus k$	1. DEM	2. FRF	3. GBP	4. JPY	5. USD
1. DEM	+1.00	+0.05	−0.79	+0.76	−0.87
2. FRF	+0.05	+1.00	+0.35	−0.30	−0.26
3. GBP	−0.79	+0.35	+1.00	−0.93	+0.61
4. JPY	+0.76	−0.30	−0.93	+1.00	−0.74
5. USD	−0.87	−0.26	+0.61	−0.74	+1.00

quite different from time series $RVal_{i5}(t/1)$ and $RVal_{i1}(t)$ using the U.S. dollar or the German mark as the base currency, respectively. Again, it is important to observe that the time series of $ICVI$ $RNVal_i(t/1)$ for each currency would be the same no matter which of the currencies is used as numeraire (or base currency). Now it is possible to calculate correlation coefficients *for all pairs* of time series $RNVal_i(t/1)$, $i=1,\ldots,5$.

Table 2 reports these correlation coefficients for the 1981–1998 period. As shown there, a close association exists between the U.S. dollar (USD) and British pound (GBP), but these currencies were negatively correlated with the European (DEM, FRF) and Japanese (JPY) currencies. Notice that these empirical relationships cannot be observed by investigation of time series $Val_{ij}(t)$ (or $RVal_{ij}(t/1)$), $i=1,\ldots,5$, $t=1,\ldots,216$, where the U.S. dollar (panel A of Table 1) or the German mark is the base currency (panel B of Table 1).

The economic meaning of $ICVI$ $NVal_i(t)$ (or $RNVal_i(t/t_0)$ is that changes in value of the corresponding currencies can be assessed in the context of an average. For example, statements about the value of the U.S. dollar depend on the base currency (i.e., yen, pounds, euros, etc.). The dollar could be rising (appreciating) against the yen but decreasing (depreciating) against the pound sterling. By contrast, if the $ICVI$ of the dollar was increasing, we could infer that the value of the dollar was rising on average against the currencies used in the computation of the geometric mean currency value. $ICVI$ using major hard currencies would provide an index of average movement in the value of the dollar in world currency markets. This index would be valuable in assessing the dollar's value in a global perspective and, therefore, would be useful to investors, government, and others.

4. Minimum variance currency basket

We next demonstrate the application of our invariant currency value index ($ICVI$) to the problem of solving for a *minimum variance currency basket* ($MVCB$). Our purpose is to solve for a set of optimal weights for different currencies. A key problem here is that the optimal currency weights are dependent on the base currency chosen by the investigator. The conception of $ICVI$ enables us to avoid this problem because normalized index $NVal_i(t)$ (or $RNVal_i(t/t_0)$) of ith currency's value is invariant to the base currency choice; as such, optimal weights are obtained with a precision of a positive homogeneous transformation.

Our minimum variance basket of currencies has economic significance in terms of its potential use as a numeraire. Commenting on the notion of a currency (or commodity) numeraire, Adam Smith stated in the 18th century: "... a commodity which is itself continually varying in its own value, can never be an accurate measure of the value of other commodities." (Smith, 1976, p. 48). Despite the fact that it is impossible to find a universally traded good with constant exchange value "across time, space, and economic systems" (Seton, 1992), there have been repeated attempts to establish such a standard, including the construction of aggregate currencies comprised of a basket of different currencies. [6]

Rather than seeking a numeraire that is ideally constant over time, in this section we propose an alternate goal of constructing a low-volatility (i.e., low-risk), multi-currency numeraire for a fixed set of currencies and for a fixed period of time. More specifically, we propose to construct an index of value in exchange in the form of the weighted arithmetical mean, or

$$Ind(w;t) = \sum_{i=1}^{n} w_i RNVal_i(t/t_0) \tag{4.1}$$

of reduced normalized values in exchange $RNVal_i(t/t_0)$, $i=1,\dots,n$, $t=1,\dots,T$, of the goods $g_1,\dots,g_n$, wherein this index is determined by a weight vector $w=(w_1,\dots,w_n)$ (i.e., vector of non-negative weight coefficients $w_1,\dots,w_n$, $w_1+\cdots+w_n=1$).

For interpretation of the formally introduced index $Ind(t;w)$, we will use the notion of *aggregate currency* $AC(q_1,\dots,q_n)$ (more generally—*aggregate good* $AG(q_1,\dots,q_n)$) defined as a basket of simple currencies (goods), taken from a fixed set $G=\{g_1,\dots,g_n\}$ in fixed amounts $q_i>0$, $i=1,\dots,n$:

$$AC(q) = AC(q_1,\dots,q_n) = \{q_1[u_1],\dots,q_n[u_n]\}. \tag{4.2}$$

Here $q=(q_1,\dots,q_n)$ is a vector of the currencies' amounts.

Let us suppose that a *value in exchange of a basket* $AC(q)$ of simple currencies is determined by the sum

$$Val(AC(q)) = \sum_{i=1}^{n} Val(q_i[u_i]). \tag{4.3}$$

[6] Some historical examples of benchmark or numeraire commodities are: *gold and silver money* of mercantilists (Bonar, 1909; Davies, 1996), *labor* of Smith and Ricardo (Bonar, 1909; Kurz and Salvadori, 1997), *abstract labor* of Marx (Sraffa, 1960; Kurz and Salvadori, 1997), *wage unit* of Keynes, *standard commodity* and *common labor* (Sraffa, 1960; Passinetti, 1981; Krause, 1982; Kurz and Salvadori, 1997), *unit of consumption* (Seton, 1992), *ideal equilibrium price* of neo-classical economics (Walras, 1954; Debreu, 1959), *energy unit* (Georgescu-Roegen, 1976), and many others (e.g., see also Hart et al., 1964; Hart, 1966; Greenfield and Yeager, 1983; Cowen and Kroszner, 1994). Examples of well-known aggregated currencies that have been used as a regional or global numeraire include the *EUA* (European Unit of Account) until 1979, the *ECU* (European Currency Unit) since 1979, euro, *SDR* (Special Drawing Rights), *TR* (Transferable Rouble of Comecon) from 1963 to 1991, *The Economist*'s McDonald's Big Mac Hamburger Index, etc.

We should note that the form of the value function $Val(AC(q))$ in (4.3) is specific to our purposes and cannot be formally derived from our SIMEX. Broadly speaking, the value in exchange of an aggregated good may be calculated by different formulas.

For a fixed moment t one can infer from (4.3) the representation

$$Val(AC(q));t) = \sum_{i=1}^{n} q_i Val([u_i];t) = \left(\sum_{i=1}^{n} q_i c_{ij}(t)\right) Val([u_j]). \tag{4.4}$$

This equation can be readily transformed as follows:

$$\frac{Val(AC(q);t)}{Val(AC(q);t_0)} \sqrt[n]{\prod_{r=1}^{n} \frac{c_{rj}(t_0)}{c_{rj}(t)}} = \sum_{i=1}^{n} w_i RNVal_i(t/t_0), \tag{4.5}$$

where weight-coefficients $w_1,\dots,w_n$, $w_i \geqslant 0$, $w_1+\cdots+w_n=1$, are determined by the expression

$$w_i = \frac{q_i c_{ij}(t_0)}{\sum_{r=1}^{n} q_r c_{rj}(t_0)}. \tag{4.6}$$

Comparison of (4.5) with (4.1) yields the index of the basket's value in exchange

$$Ind(w;t) = \frac{Val(AC(q);t)}{Val(AC(q);t_0)} \sqrt[n]{\prod_{r=1}^{n} \frac{c_{rj}(t_0)}{c_{rj}(t)}}. \tag{4.7}$$

From (4.7) we see that index $Ind(t;w)$ may be treated as a *reduced* (to the moment t_0) *normalized* (i.e., divided by corresponding geometric mean) value in exchange of the aggregate currency $AC(q)$ for the moment t. Such an interpretation of index $Ind(t;w)$ justifies a rather cumbersome designation $Ind(t;w) = RNVal(AC(q);t/t_0)$.

We next construct a *minimum variance currency basket* $(MVCB)AC(q^*)$ for a fixed set of currencies, given time series $c_{ij}(t)$, $i,j=1,\dots,n$, and fixed period of time $[1,T] = \{1,2,\dots,T\}$.

The volatility of the currency invariant index time series can be measured by the variance:

$$S^2(w) = \mathrm{var}(w) = \frac{1}{T}\sum_{t=1}^{T} [Ind(w;t) - MInd(w)]^2, \tag{4.8}$$

where

$$MInd(w) = \frac{1}{T}\sum_{t=1}^{T} Ind(w;t) \tag{4.9}$$

is the arithmetical mean of the corresponding values in exchange. The optimal weight-vector $w^* = (w_1^*,\dots,w_n^*)$ is determined by minimizing the variance $S^2(w) = \mathrm{var}(w)$ under the constraints:

$$w_i \geqslant 0,\ \ i=1,\dots,n, \quad w_1+\cdots+w_n = 1. \tag{4.10}$$

The variance $S^2(w)$ may be represented as

$$S^2(w) = \sum_{i,k=1}^{n} w_i w_k \operatorname{cov}(i,k) = \sum_{i=1}^{n} w_i^2 s_i^2 + 2 \sum_{\substack{i,k=1 \\ i<k}}^{n} w_i w_k \operatorname{cov}(i,k), \tag{4.11}$$

where $\operatorname{cov}(i,k)$ is the covariance of time series $RNVal_i(t/t_0)$, $RNVal_k(t/t_0)$, and s_i^2 is the variance of the time series $RNVal_i(t/t_0)$, $i,k = 1,\dots,n$, $t = 1,\dots,T$. Thus, the optimization problem is reduced to minimizing the quadratic form $S^2(w)$ in (4.11) under the linear constraints in (4.10).[7] There are a variety of numerical methods to solve the optimization problem stated in (4.10) and (4.11). Here we use Newton's method, which is computed using the application Solver.xla for Microsoft Excel 7.0.

The optimal amounts $q_1^*,\dots,q_n^*$ of the currencies, which are contained in the optimal aggregate currency $AC^* = AC(q^*)$, can be calculated as a system of linear equalities. Recalling (4.6), we can write

$$q_i = \frac{w_i}{c_{ij}(t_0)} \mu, \tag{4.12}$$

where μ is an arbitrary positive constant (e.g., for $\mu = 1$, one gets the equality Val (1 unit of SAC; t_0) $= Val$ (1 unit of currency j; t_0). Therefore, any vector $q^* = (q_1^*,\dots,q_n^*)$ with components that are proportional to the numbers $w_i/c_{ij}(t_0)$, $i = 1,\dots,n$, is optimal and determines an optimal aggregate currency $AC^* = AC(q^*)$ and time series $Ind^*(t) = Ind(w^*;t) = RNVal(AC^*;t)$, $t = 1,\dots,T$ of minimal variance. Hereafter, we will refer to the currency basket $AC^* = AC(q^*)$ as a *stable aggregate currency* (SAC).

5. Application to hardening the *SDR*

In this section we compute the content of SAC based on the same five currencies used in the International Monetary Fund's (IMF) Special Drawing Rights (SDR) and comparatively examine their relative stability over time.

The SDR is an international reserve asset that was proposed by the 'Group of Ten' and adopted by the International Monetary Fund (IMF) in 1969. It is used by the IMF to denominate its financial activities (e.g., loans to its more than 180 member countries) and as an international reserve unit for member countries. The SDR has a variety of international monetary roles, including facilitating international trade and finance, providing a translational unit of account for denominating business transactions,

[7] Obviously, this framework is distinct from the well-known approach taken in modern portfolio theory (see Markowitz, 1959; Sharpe, 1970), where minimization of a quadratic form (variance of a portfolio's random return) is conditioned by a fixed mathematical expectation of the random return. Additionally, our conception of minimum variance currency basket ($MVCB$) is distinct from the Rustem's "multiple currency version of quadratic mean-variance portfolio optimization" (see Rustem, 1995, p. 907). In the multiple currency version of portfolio optimization "the resulting portfolio takes account of the return risks in investments, in a currency, and the currency risk, with respect to a *base currency* [emphasis added]" (see Rustem, 1995, p. 907). However, the main idea of a minimal variance basket is quite different—we prefer not to use an arbitrarily selected base currency (i.e., a change of a base currency would alter Rustem's optimal portfolio), but instead construct a base currency as a currency basket of minimal risk (variance).

anchoring currency values in pegging operations of monetary authorities, and proxying a constant value for international legal conventions. Also, some commercial banks create private *SDR*s on demand for their customers. However, as Coats (1989, p. 17) has observed, the *SDR*'s use as a unit of account has been limited due to its non-constant purchasing power or value over time. The weights for the *SDR* currencies are calculated on the basis of exports of goods and services of each member country and the balances of these currencies held by all IMF member countries. Currencies' weights are revised based on exchange rate data in the last three months of a review year and then remain fixed for the forthcoming 5-year period. The IMF's executive review board can revise the currency weights more often if it desires but historically this option has not been exercised. Importantly, the *SDR* is intended to provide a stable unit of account for purposes of international trade and finance.

Over the years there has been considerable discussion by monetary authorities to 'harden' (or stabilize) the *SDR* due to the IMF's desire to provide an international yardstick that would promote monetary stability (e.g., Coats, 1989; Thakur, 1994; Mussa et al., 1996). As Thakur (1994) has pointed out, "The standard basket's value … is not stable in terms of individual currencies. Thus, an appreciation (depreciation) of any one currency in the basket in terms of the other currencies raises (lowers) the value of the *SDR* in terms of those other currencies." (Thakur, 1994, p. 2).

We next compute a *SAC* comprised of the same currencies in the *SDR*. We find that *SAC* is both stable over time and has little (or no) correlation with the individual currencies in the basket. As such, an appreciation (depreciation) of any constituent currency would not raise (lower) the value of the *SAC* for the most part. We infer that the optimized currency baskets could be useful in IMF efforts to harden the *SDR*.

For time series $RNVal_i(t/1)$, $i = 1, \dots, 5$, $t = 1, \dots, 216$, using monthly observations for the period January 1981 to December 1998, the calculated (i.e., using Solver.xla for Microsoft Excel 7.0.) optimal weight coefficients for the five currencies *DEM*, *FRF*, *GBP*, *JPY*, and *USD*, respectively, are

$$w_1^* = 0.2595; \quad w_2^* = 0.1155; \quad w_3^* = 0.3259;$$
$$w_4^* = 0.1265; \quad w_5^* = 0.1726. \tag{5.1}$$

These optimal weights can be transformed (see (4.12)) into optimal amounts $q_1^*, \dots, q_n^*$ of the currencies, which constitute the minimum variance currency basket (*MVCB*), i.e., stable aggregate currency:

$$SAC = \{0.68[DEM], 0.70[FRF], 0.17[GBP], 32.21[JPY], 0.21[USD]\}. \tag{5.2}$$

The constructed *SAC* can readily be used in payments of goods and services, payment of interest and principal on debt contracts, etc. Also, we can obtain exchange rates such as *USD*/*SAC*, *EUR*/*SAC*, *JPY*/*SAC*, etc. As such, daily exchange rates can be used to convert local prices into *SAC* prices over time (e.g., stock prices in local currency for any given day can be converted to *SAC* prices that contain little or no exchange rate risk).

As shown in Table 3, after reducing the time series *SAC* ($t = 1, \dots, 216$) to one at $t = 1$, we see that *SAC* has a small standard deviation s^* in comparison with standard

Table 3
Standard deviations $s_1, \ldots, s_5$ of five simple currencies (German mark (*DEM*), French franc (*FRF*), British pound (*GBP*), Japanese yen (*JPY*), U.S. dollar (*USD*)), and two aggregated currencies (*SDR* and *SAC*) calculated on monthly data for the period 1/31/81–12/31/98:

Currency	St.d.
DEM	0.0906
FRF	0.0510
GBP	0.0950
JPY	0.2669
USD	0.1644
SDR	0.0320
SAC	0.0048

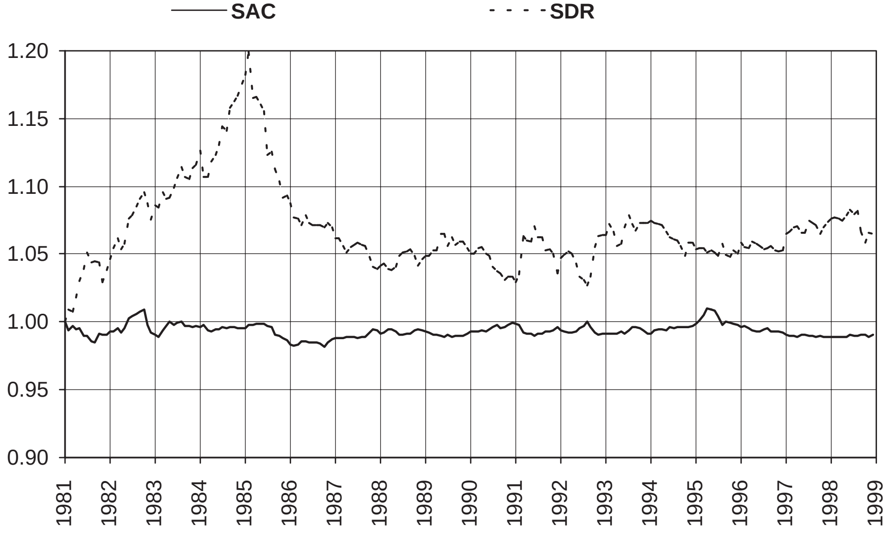

Fig. 1. Dynamics of reduced normalized values in exchange of the *SAC* and of *SDR* (based on monthly data for the period 1/31/81–12/31/98).

deviations $s_1, \ldots, s_5$ of time series $RNVal_i(t/1)$, $i = 1, \ldots, 5$, $t = 1, \ldots, 216$ corresponding to the simple (non-aggregate) currencies. Notice that the volatility (measured by standard deviation $s^* = 0.0048$ of the corresponding time series) of the optimal aggregate currency is about 10 times smaller than the standard deviation of simple currency *FRF* with minimal standard deviation $s_2 = 0.051$. Furthermore, our optimal basket of currencies has a standard deviation that is about seven times smaller than the standard deviation $s_S = 0.032$ of aggregate currency *SDR*, which is known for its small volatility (see Redhead and Hughes, 1988). The much lower volatility of our optimal currency basket $AC^* = SAC$ compared to the *SDR*'s volatility is graphically shown in Fig. 1.

Table 4 reports the correlation coefficients for pairs of time series $RNVal_i(t/1)$, $i = 1, \ldots, 5$ corresponding to the simple currencies' (*DEM*, *FRF*, *GBP*, *JPY*, *USD*)

Table 4
Correlation coefficients of five simple currencies (German mark (*DEM*), French franc (*FRF*), British pound (*GBP*), Japanese yen (*JPY*), and U.S. dollar (*USD*)) with two aggregated currencies (*SDR* and *SAC*) calculated using monthly data for the period 1/31/81–12/31/98

$i \setminus k$	*SAC*	*SDR*
1. *DEM*	+0.05	−0.57
2. *FRF*	+0.09	−0.60
3. *GBP*	+0.05	+0.12
4. *JPY*	+0.02	−0.29
5. *USD*	+0.03	+0.83
SDR	+0.10	+1.00
SAC	+1.00	+0.10

time series, as well as aggregated currencies *SDR* and *SAC*. It is clear from this information that fluctuations of aggregate currency *SDR* are highly positively correlated with fluctuations in *GBP* and *USD* but negatively correlated with *DEM*, *FRF*, and *JPY*. By contrast, fluctuations in *SAC* are (for all practical purposes) uncorrelated with fluctuations in the simple currencies as well as with fluctuations of the aggregated currency *SDR*.

This low correlation property of *SAC* is particularly important with respect to the problem of hardening the *SDR*. Recall that Thakur (1994) has noted that a 'harder' standard basket of currencies must be stable in terms of fluctuations in the values of individual currencies contained in the basket.

The notion of minimum variance currency basket (*MVCB*) can be applied to any of the aforementioned proposals for a currency numeraire, the construction of a currency basket for purposes of currency pegging operations (e.g., as of April 1996 three members of the IMF pegged their currencies to the *SDR*), and hardening the *SDR* in terms of lowering its volatility and correlation with component currencies over time. It is reasonable to presume that the currency weights derived from minimum variance methods could be adjusted by the IMF, World Bank, government representatives, or other parties involved in the final construction of a currency basket to take into account economic trade, country size, etc. These adjustments are beyond the scope of the present paper.[8]

6. Application to world money

Related to the idea of aggregated currencies, there have been repeated proposals for a world currency ('world money'). Nobel Prize economist Professor Robert Mundell (2000c), who is best known for his pioneering work with respect to the euro, has been

[8] Considerable research has been published on the subject of obtaining optimal currency weights in the context of economic policy goals, including export trade goals (e.g., see Branson and Katseli Papaefstratiou, 1980; Lipschitz and Sundararajan, 1980, 1982; Edison and Vardal, 1985) and real exchange rates (e.g., see Edison, 1986).

a leading proponent of the notion of world money.[9] Mundell (2000a) has proposed a G-3 world currency with weights of 45% dollars, 20% yen, and 35% euros. Ogawa and Ito (2000) have proposed a similar three-currency world currency. According to Mundell, this G-3 basket currency would not be much different from Special Drawing Rights (*SDR*), which is comprised now of the dollar, yen, pound, and euro (which has replaced the mark and franc). Such a basket would represent a *world currency* but countries would retain the use of the home currencies for domestic purposes.[10]

In recent papers and speeches Mundell (2000a–c) has recommended that a world currency based on a basket of hard currencies (e.g., the dollar, euro, and yen) could be a key component of a global economy. A world currency would provide a single numeraire that would help unify capital, goods, and services into a global market linking the Americas (or NAFTA countries), the European Union, and Asia, as well as their trading partners in other countries. While international monetary reform in terms of *world money* has long been an aspiration of governments throughout history, political and economic realities today make the possibility of a universal unit of account more plausible than ever (Mundell, 2000b, p. 338; 2000c, p. 6).

We agree with Professor R. Mundell, that an aggregate world currency may be more stable than any currency that is included in the corresponding basket. In his words, "a multiple-currency basket does not suffer from the possible defect of a single-currency basket, namely, that the currency appreciates (or depreciates) significantly against other currencies." (Mundell, 2000a, p. 12). Here we seek to investigate Professor Mundell's proposition concerning the composition of the currency basket: "One possibility would be a basket of the three currencies, with weights of 45% dollar, 20% yen, and 35% in euros. That would be a pretty good basket for the whole world economy for the next few years. … That three-currency basket could be a good unit of account." (Mundell, 2000a, p. 12). We will symbolize this currency basket using the abbreviation *MIM* (Mundell's International Money), which is determined by the three-currencies basket $MIM = \{35\%\ EUR,\ 20\%\ JPY,\ 45\%\ USD\}$. We next compare *MIM*'s volatility with the volatility of our SAC composed of EUR, JPY, and USD.

IMF statistical series for euro, yen and dollar daily exchange rates are gathered from the IMF's website for the period January 1, 2000, to December 31, 2000. First, we compute the three time series of reduced (to the moment $t=1$: January 1, 2000) normalized

[9] Mundell has noted that there was a plan for a world currency based on gold coins at the 1867 Paris conference, and other plans in post-WWII years by the British based on Keynes' 'bancor' and by the Americans for the 'unitas'. In response to restructuring of monetary systems after WWI, Van de Putte (1920) proposed to the League of Nations the "dolli" based primarily on the U.S. dollar. Fisher (1913, pp. 501–510) commented on the need for a universal unit of account for purposes of purchasing goods and services; likewise, Shiller (1998, p. 33) has observed that countries could easily compute indexed units of account for their citizens (see also Shiller, 1993; Coats, 1994).

[10] Ho (2000) has also proposed a standardized world currency basket for use by countries using a currency board arrangement and a global indexed bond to standardize debt instruments in different countries. Finally, a private organization, the Centre Jouffroy Pour la Réflexion Monétaire in Paris, France, has proposed a 'new bancor' that is comprised of the dollar and euro in equal parts due to similarities in US and EU population, GDP, trade, size, etc.

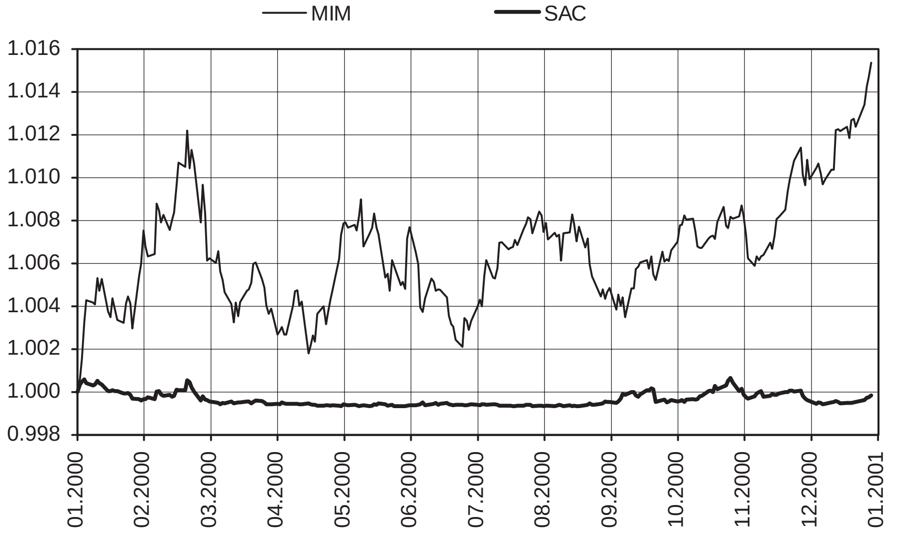

Fig. 2. Dynamics of reduced normalized indices of value in exchange of the *SAC* and *MIM* (based on daily data for the period 1/1/00–12/31/00).

Table 5
Standard deviations and coefficients of correlation for three simple currencies (*EUR*, *JPY*, *USD*) and two aggregated currencies (*MIM*, *SAC*) calculated using daily data for the period 1/1/00–12/31/00

	EUR	*JPY*	*USD*	*MIM*	*SAC*	St.d.
EUR	+1.00	−0.85	−0.90	−0.20	+0.01	0.0327
JPY	−0.85	+1.00	+0.53	−0.34	+0.02	0.0176
USD	−0.90	+0.53	+1.00	+0.61	+0.01	0.0222
MIM	−0.20	−0.34	+0.61	+1.00	+0.12	0.0026
SAC	−0.01	+0.02	+0.01	+0.12	+1.00	0.0003

indices of value in exchange $RNVal_i(t/1)$, $i = 1(EUR), 2(JPY), 3(USD)$, $t = 1,\ldots,T = 1,\ldots,259$ (trading days). Second, we obtain optimal weights for the minimum variance basket of currencies: $SAC = \{34.74\%\ EUR,\ 33.49\%\ JPY,\ 31.77\%\ USD\}$, which differ from the weight scheme for $MIM = \{35\%\ EUR,\ 20\%\ JPY,\ 45\%\ USD\}$. Third, we compute the two time series of reduced (to the moment $t = 1$: January 1, 2000) normalized indices of value in exchange $SAC(t/1) = RNVal(SAC; t/1)$, $MIM(t/1) = RNVal(MIM; t/1)$, $t = 1,\ldots,T = 1,\ldots,259$. Fig. 2 graphs these two currency basket values for the sample period. Table 5 reports the standard deviations and coefficients of correlation for simple (*EUR*, *JPY*, *USD*) and aggregate currencies (*SAC*, *MIM*). As shown in Table 5, not only is *SAC* far more stable (i.e., smaller standard deviation) than *MIM*, but also *SAC* is practically uncorrelated with the simple currencies (*EUR*, *JPY*, *USD*).

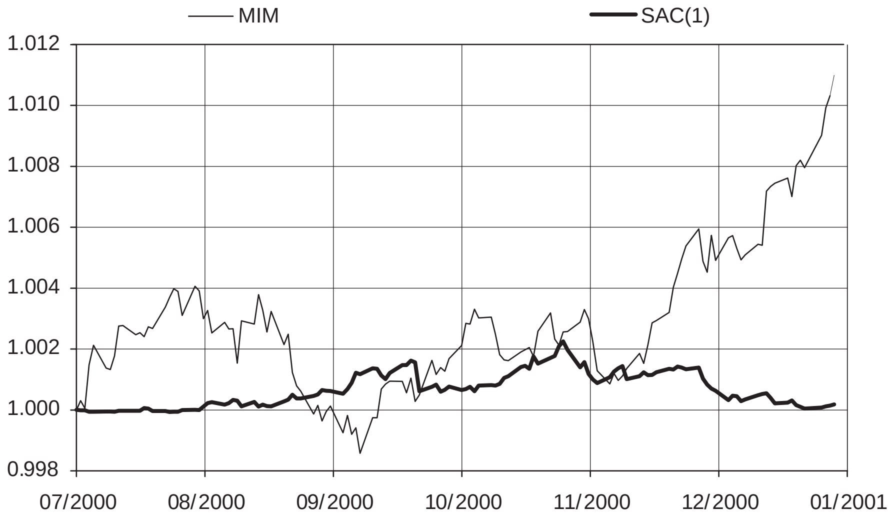

Fig. 3. Dynamics of reduced normalized indices of value in exchange of $SAC1$ and MIM (based on daily data for the period 7/1/00–12/31/00).

It should be recognized that the negligibly small volatility of SAC is achieved using historical data series. Professor Mundell formed MIM's basket ex ante facto (i.e., before the three currencies' time series were observed in reality); by contrast, SAC's basket is formed ex post facto (i.e., after all data for the three time series under investigation are observed).

We next test the SAC's performance under ex ante facto conditions. For this purpose we divide the discrete total time interval under investigation $[1, T]$, $T = 259$ into two nearly equal disjoint discrete time sub-intervals $[1, T_1 - 1]$ and $[T_1, T]$, where $T_1 = 131$ (the date July 1, 2000). Three time series of reduced (to the moment $t = 1$: January 1, 2000) normalized indices of value in exchange $RNVal_i(t/1)$, $i = 1(EUR), 2(JPY), 3(USD)$, $t=1, \ldots, T_1-1=1, \ldots, 130$ are calculated, and the optimal (for the first time period $[1, T_1 - 1]$) basket is formed: $SAC1 = \{32.96\%\ EUR,\ 33.93\%\ JPY,\ 33.11\%\ USD\}$. This optimal basket is now employed in the second discrete time interval $[T_1, T]$, $T_1 = 131$, and $T = 259$ (the date December 31, 2000). Three time series of reduced to the moment $T_1 = 131$ normalized indices of value in exchange $RNVal_i(t/T_1)$, $i = 1(EUR), 2(JPY), 3(USD)$, $t = T_1, \ldots, T$ are calculated. Two time series of reduced to the moment $T_1 = 131$ (the date July 1, 2000) normalized indices of value in exchange $SAC1(t/T1)$ and $MIM(t/T1)$, $t = T_1, \ldots, T$ are calculated. Fig. 3 graphs the results over time for these two ex ante currency baskets. Table 6 gives standard deviations and coefficients of correlation for simple (EUR, JPY, USD) and aggregate ($SAC1$, MIM) currencies calculated for the second discrete time interval $[T_1, T]$. The results show that ex ante $SAC1$ (constructed using data of the first time period $[1, T_1 - 1]$) is far more stable during the second time period $[T_1, T]$) than ex ante MIM. Of course, as shown in Table 6, the ex ante basket currency $SAC1$ is somewhat less stable than the

Table 6
Standard deviations and coefficients of correlation for three simple currencies (*EUR*, *JPY*, *USD*) and three aggregated currencies (*MIM*, *SAC*1, *SAC*2) calculated using daily data for the period 7/1/00–12/31/00

	EUR	*JPY*	*USD*	*MIM*	*SAC*1	*SAC*2	St.d.
EUR	+1.00	−0.88	−0.77	+0.37	−0.95	+0.01	0.0250
JPY	−0.88	+1.00	+0.38	−0.76	+0.82	+0.01	0.0177
USD	−0.77	+0.38	+1.00	+0.31	+0.78	+0.01	0.0136
MIM	+0.37	−0.76	+0.31	+1.00	−0.28	+0.07	0.0024
*SAC*1	−0.95	+0.82	+0.78	−0.28	+1.00	+0.28	0.0006
*SAC*2	+0.01	+0.01	+0.01	+0.07	+0.28	+1.00	0.0002

ex post basket currency *SAC*2, which is optimized ex post facto on the second time interval $[T_1, T]$: $SAC2 = \{34.45\%\ EUR,\ 33.38\%\ JPY,\ 32.17\%\ USD\}$. Nonetheless, it is clear that ex ante *SAC* is still a viable candidate for stable world money. As shown in Table 6, however, correlation coefficients for *SAC*1 are like those for single currencies.

As already mentioned, Mundell (2000a) has proposed that a world currency constructed from an indexed basket of G-3 currencies could be used to establish a world unit of account. Named the 'intor' (to connotate 'international' and the French word for gold), the IMF and G-3 countries could establish fixed exchange rates between each currency and the intor using a currency-board system. Each country would continue to have its own domestic currency to retain powers over seigniorage. We believe that minimum variance currency baskets such as *SAC* could offer a world money that is (in Milton Friedman's words) "… impersonal and not subject to the control of political authority …" which was a major reason for the use of gold as a pseudo world money in the 19th and early 20th centuries (Mundell and Friedman, 2001, p. 28).

7. Conclusions and implications

We provide an exact and computable solution for the problem of an index (indicator) of a currency's value in exchange, the index being independent of base currency choice. In brief, our *invariant currency value index* (*ICVI*) is computed by using the geometric mean of all currencies taken into account to normalize the currency's value. Thus, given a fixed set of currencies, the invariant index of a currency from the set will have the same value, regardless of base currency choice. The methodological and mathematical reasoning behind the invariant index of currency's value in exchange is formulated in the framework of a *simple exchange model* (SIMEX). This model takes into account only pair-wise direct exchange of any two goods (e.g., commodities, services, currencies, etc.).

To demonstrate one possible application we employ our *ICVI* to construct a *minimum variance currency basket* (*MVCB*). Utilizing a variance minimization framework, we compute optimal weights for currencies and construct a *stable aggregate currency* (*SAC*) comprised of five different hard currencies (i.e., German mark, French franc, British pound, Japanese yen, and U.S. dollar). Comparative empirical analyses of our five-currency *SAC* to the IMF's Special Drawing Rights (*SDR*) for the period

1981–1998 demonstrates that SAC has lower volatility than the SDR as well as its component currencies. Consistent with Adam Smith's observation, the best numeraire for valuing commodities (and currencies) is a commodity (or currency) that is stable over time. In this regard, based on our analyses, we conclude that SAC could be used to harden the SDR.

Additionally, SAC has potential implications to the development of a world currency as proposed by Nobel Laureate Professor Mundell and others. World currency proponents have primarily focused on the international policy and structural reforms needed to implement a basket currency, with little or no attention to the mathematical construction of the basket currency index. SAC could be valuable to computational discussions in this area. As an example, we consider Professor Mundell's recommended currency basket comprised of the dollar, euro, and yen. Comparative analysis of Mundell's recommended world currency basket to our minimum variance currency basket constructed from the same currencies confirms the low volatility and correlation properties of SAC. We conclude that SAC could be used to develop a world money that would be easy for businesses in the financial services industry to implement, as well as impersonal and not subject to political authority.

Finally, as pointed out by an anonymous referee, a major implication of our invariant currency value index and stable aggregate currency is their contribution to the construction of index numbers for contract settlement. As observed by Shiller (1993, p. 209), while consumer price indices, real estate valuation indices, stock price indices, and many others began as theoretical constructs of interest to a small audience of interested economists, they later became widely accepted by the public as common tools for establishing settlement claims in risk management contracts. Emphasizing the importance of indices, he commented that "A major barrier to the establishment of many macro markets today is a shortage of indices that are agreed upon as well suited to be the basis of cash settlement of contracts." (Shiller, 1993, p. 208). In the present paper $ICVI$ is an index of value that eliminates the problem of base currency. Also, SAC is a tool for constructing minimum variance currency baskets.

We believe that these new indices contribute to the effort to create widely agreed upon measures of international currency, as well as in analyses involving international prices of financial assets, real goods, and services. We infer that optimal currency baskets have potential applications to a number of academic and practical issues in business, including exchange rate risk measurement, portfolio analysis of multi-currency assets, international capital budgeting decisions with multi-currency cash flows, the practice of reporting constant currency accounting numbers on financial statements, etc. Future research is recommended to investigate these and other potential applications as well as to increase our understanding of currency invariant indices and minimum variance currency basket indices.

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Appendix. Further discussion

In this Appendix we seek to provide further discussion concerning the interpretation of our currency invariant index ($NVal$) and stable aggregate currency index (SAC). Our discussion is intended to address alternative views of our methodology, including the comment by Siegmann in this issue. Before doing so, it is worthwhile to reiterate our research problem—namely, we seek to solve for a minimum variance basket currency that is not dependent on base currency choice.

We are aware that some readers may well interpret $NVal$ to be a special case of an effective exchange rate index (EER). The Bank of England, IMF, and others construct trade-weighted EERs to measure currency values. While it may be perceived that our $NVal$ is simply an equal weighted EER, this interpretation is incorrect. As an example, suppose that we use the US dollar, yen, and euro to compute an EER for the dollar. Using equal weights, the effective exchange rate for the dollar would be $(YEN/USD \times EURO/USD)^{1/2}$. However, this value differs from our $NVal(USD)$ equal to $(USD/USD \times YEN/USD \times EURO/USD)^{1/3}$. Notice that the root in the effective exchange rate calculation is related to the number of exchange rates (i.e., two exchange rates), while $NVal(USD)$ is related to the number of currencies (i.e., two exchange rates are formed from three currencies). Thus, $NVal_i$ is calculated as a geometric mean of n exchange coefficients $c_{i1}, \ldots, c_{in}$, *including* $c_{ii} = 1$, whereas EER_i is calculated as a geometric mean of $(n-1)$ exchange coefficients $c_{i1}, \ldots, c_{i-1i}, c_{i+1}, \ldots, c_{in}$, *excluding* $c_{ii} = 1$.

It is important to note that, if you put trade weights on currencies or exchange rates (as in the typical EER), currency invariance will no longer hold. In our previous example, given that $EER(USD) = (YEN/USD \times EURO/USD)^{1/2}$, we can alternatively write $EER(USD) = (YEN/USD)/(YEN/YEN \times YEN/EURO)^{1/2}$, where the yen is the base currency (i.e., compared to our $NVal$ notice that YEN/USD is not included in the denominator and the 1/2 root rather than the 1/3 root is used), or $EER(USD) = (EURO/USD)/(EURO/YEN \times EURO/EURO)^{1/2}$, where the euro is the base currency (i.e., compared to our $NVal$ notice that $EURO/USD$ is not included in the denominator and the 1/2 root rather than the 1/3 root is used). All three of these $EER(USD)$ values need to be equal to one another in order for currency invariance to hold. Equal weights on the exchange rates in the denominators of all three equations would yield this result. But how would unequal trade weights affect the results of these equations? It should be immediately obvious that, because the weights are determined by trade flows between countries (and not a desire to obtain currency invariance), the dollar values computed from these three equations will differ and, therefore, currency invariance does not hold for a typical *EER*.

We can infer from the above discussion that our currency invariant index *NVal* has some similarities to effective exchange rates but is uniquely different in some crucial ways. While *EER*s have $n-1$ trade weights for n currencies, *NVal* has n equal weights for n currencies. Also, *EER*s using trade weights violate the currency invariance property. As such, depending on the base currency applied, *EER*s would generate different optimal weights in solving for a minimum variance currency basket of currencies. For this reason we cannot use *EER*s to solve our research problem of finding the minimum variance currency basket.

It is interesting to consider the application of our indices to the problem of computing a minimum variance portfolio of stocks. Suppose we seek to build a minimum variance portfolio comprised of the following common stocks: Microsoft (MSFT), Proctor and Gamble (PG), and Exxon (XOM). If we quote our stock prices in dollars, then our data is as follows: USD/one share of MSFT, USD/one share of PG, and USD/one share of XOM. That is, we have three exchange rates and four 'currencies' in this problem. So, we should get four optimal weights for our portfolio, where the fourth asset is the US dollar. Notice that, if we used the effective exchange rate, we would get three optimal weights for the three exchange rates (instead of four weights). Effective exchange rates would not count the dollar as a fourth asset.

Before proceeding further with this example, we should mention that an American buying only US stocks in US dollars has no exchange rate risk. As such, the US dollar has mean return and standard deviation equal to zero. In this case the minimum variance basket is all dollars (cash), which is well-known by researchers. Most individuals would infer as much from common sense.

Alternatively, if base currency choice is a problem in our stock portfolio problem, then our indexes could be useful. If you apply US dollars versus Japanese yen as the base currency to quote stock prices, you will get two different series of stock returns and, in turn, two different sets of optimal weights on the minimum variance stock portfolio. Which stock portfolio is the optimal one? Other base currencies will further confuse the issue. What is the optimal stock portfolio for a large institutional investor that is global in scale with investments spanning international stock markets? We solve the problem of base currency by constructing our currency invariant index *NVal*, which can be used to compute *SAC* for some set of currencies. Stock prices can be expressed in *SAC* by converting say dollar prices to *SAC* prices using the *SAC*/dollar exchange rate. Now stock returns have no local currency risk—i.e., the local return is the *SAC*-based return plus the local currency return per unit of *SAC*. All investors around the world earn the *SAC*-based return; however, investors' actual returns will differ depending on their local currency. In this situation the set of optimal weights obtained for the minimum variance stock portfolio using *SAC*-based returns is independent of local currency choice. This optimal portfolio based on *SAC* can be compared to the optimal portfolio based on dollars (or yen, euros, etc.) to examine how an investor's local currency—i.e., the dollar (yen, euro, etc.)—contributes to their portfolio diversification. Extending this concept, one could examine the degree of diversification achieved by the large institutional investor from currency risk versus non-currency risk (e.g., country risk) when investing in stock markets around the world. And, *SAC*/dollar exchange rates could be employed to study the relationship between dollar exchange risk and stock returns. These investment implications of *SAC* are intended to help demonstrate its economic meaning. We believe there is many other potential uses of *SAC* in financial economics.

Finally, we are aware that some readers may have difficulty in interpreting the meaning of *SAC* in monetary terms. In Eq. (5.1) of our paper, we show the optimal weights for five currencies. However, if one wants to know the actual monetary value of a *SAC*, it is necessary to compute the weights denoted q_i^* in our paper. In Eq. (5.2) we develop a *SAC* consisting of 0.68 marks, 0.80 francs, 0.17 pounds, 32.21 yen, and 0.21 dollars. It is a simple matter to construct this *SAC* with actual currencies. For example, *SAC* contains 0.21 dollars or 21 cents. Similarly, the quantities of other currencies in one *SAC* correspond to the q_i^* weights. Over time, as the values of these currencies change against one another, this combination of currencies will stay relatively stable in value from period to period. If the dollar falls relative to the *SAC*, but the yen rises, their movements would tend to cancel each other, all else the same. This diversification principle works so long as currencies move systematically over time relative to one another. Hard currencies tend to follow systematic patterns due to purchasing power and interest rate parity conditions. To the extent that soft currencies do not behave systematically relative to one another over time, a *SAC* comprised of these currencies will have a higher variance than a *SAC* developed from hard currencies.

In sum, our main goal is to construct a unit of value with minimal variance for a fixed market of goods (e.g., currencies) and for a fixed period of time that is independent of base currency choice. Our primary interest is finding the minimize variance currency basket, as opposed to a basket with minimum correlation with other currencies. From this point of view, Table 6 shows that our *SAC* successfully passed a concrete out-of-sample test: the standard deviation of *SAC*1 (constructed ex ante facto) exceeds the standard deviation of *SAC*2 (constructed ex post facto), but the standard deviation of *SAC*1 is still 22 times smaller than any considered ordinary currency.

Of course, it is not possible to prove or disprove out-of-sample efficiency of *SAC* by one example. To test such efficiency a thorough statistical investigation must be implemented: organize continuous monitoring of exchange rates $c_{ij}(t)$ times series; recalculate routinely optimal weight coefficients (as the IMF does for the *SDR*); and elaborate statistical methods of these time series forecasting, etc., *Sapienti sat*.

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